



Technological Progress and Political Polarization

INSOOK LEE*

Bay Area International Business School, Beijing Normal University

Received: October, 2023

Accepted: June, 2024

Abstract

How does technological progress affect political polarization? For addressing this question, in an economy where taxation policy is selected by majority-rule voting and voters are differentiated by earning ability, each voter's own ideal taxation policy is obtained to measure political polarization as distance between left-wing and right-wing voters' ideal policies. Technological progress exacerbates political polarization, when it is capital-biased by increasing relative capital productivity. In contrast, technological progress does not affect political polarization, when it is unbiased by preserving relative factor productivity. Both biased and unbiased technological progresses do not affect politico-economic distortion, while only the former affects political polarization.

Keywords: Technological progress, Political polarization, Relative capital productivity, Politico-economic distortion.

JEL Classification: O33, D72, E62.

1. Introduction

In the past decades, many advanced economies witnessed growing political polarization, as they encountered various forms of technological progress. For example, in the US, as numerous technological advancements increased total factor productivity and relative capital productivity, ideological distance between left-wing and right-wing politicians significantly widened, as demonstrated by Figure 1 and 2.¹ As production technology innovation alters the way resource is allocated and distributed, individuals can change their own ideal economic policy, which may make society more polarized. It is important to understand whether and how technological progress affects political polarization. Nonetheless, to date, there is no rigorous study on the relation between technology change and political polarization. Thus, this paper analyzes the effect of technological progress on political polarization.

Understanding political polarization matters not only for political science but also for economics, because many scholars have shown that political polarization has various eco-

* ORCID ID: 0000-0002-7260-3877.

conomic bearings. For example, Alesina and Tabellini (1990) showed that political polarization leads to a rise in government debt, whereas Lindqvist and Östling (2010) found that more divided democratic society has smaller government. Azzimonti and Talbert (2014) demonstrated that political polarization amplifies business-cycle fluctuations.

To enhance understanding of political polarization, in particular, this paper contributes to the literature on the drivers of political polarization. Many scholars argued that deepening divisions in partisan politicians cause voters to be more polarized politically (e. g., Carmines *et al.*, 2012; Druckman *et al.*, 2013; Davis and Dunaway, 2016; Zingher and Flynn, 2018). However, they have not explained why politicians were more polarized in the first place. Rather, as politicians inevitably reflect the electorate, their studies do not successfully reject the reasonable case that the polarization of voters causes politicians to be more polarized. Alternatively, Gentzkow and Shapiro (2011) and Prior (2013) highlighted the role of media in increasing ideological gaps between voters but found no robust evidence. On the other hand, Garand (2010) and Duca and Saving (2016) showed that higher inequality of income makes politicians and voters more polarized. Being different from and complementing to those studies, this paper newly finds that political polarization is attributable to capital-biased technological progress. Moreover, this paper further explores whether technology-driven political polarization brings about output distortion or not.

In addition, by analyzing how technological progress changes income tax rate, this paper also contributes to the literature of tax policy responses to technology shock or change (e. g., Zhu, 1992; Chari *et al.*, 1994; Werning, 2007; Guerreiro *et al.*, 2017). Being distinct from these studies that assumed a social planner as the sole decision maker of tax policies, this paper assumes that tax policies are determined democratically by voters for analyzing political polarization.

This paper considers an economy where government taxation policies are selected by majority-rule voting and resources are allocated by competitive markets. In this economy, individual voters are differentiated by their own earning ability. While the majority-rule voting selects only one set of the government taxation policies, voters can have different sets of ideal taxation policies that maximize their own indirect utility function. Based on each voter's ideal policies, we identify the degree of political polarization with distance between ideal policies of low-earning-ability voter (left-wing voter) and high-earning-ability voter (right-wing voter).

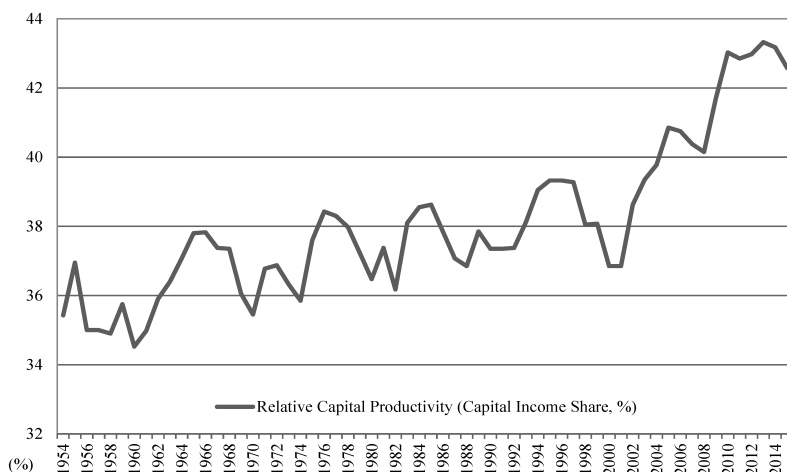
Technological progress is either biased or unbiased depending on whether relative factor productivity changes or remains the same. Biased technological progress is either capital-biased or labor-biased depending on whether relative productivity of capital or labor increases. Because an increase in the relative capital productivity means a decrease in the relative labor productivity, labor-biased technological progress is equivalent to capital-biased technological regress whose effect is opposite to the effect of capital-biased technological progress. Unbiased technological progress consists of capital-augmenting technological progress, labor-augmenting technological progress and total factor productivity improvement, since all of these preserve the relative factor productivity.

Technological progress changes voter’s ideal policies if it is biased; in contrast, it does not change any of their ideal policies if it is unbiased. In particular, when it is capital-biased, technological progress widens the distance between left-wing and right-wing voters’ ideal policies. Facing capital-biased technological progress, although they agree over the directions of the rate changes (raising capital income tax rate and lowering labor income tax rate), left-wing and right-wing voters are more divided on the ideal magnitudes of income tax rate changes. Therefore, technological progress exacerbates political polarization if it is capital-biased; and, it alleviates political polarization if it is labor-biased. In contrast, technological progress does not affect political polarization if it is unbiased. These findings highlight the importance of the relative factor productivity in the impact of technological progress on political polarization.

Furthermore, this paper examines whether such a technology-driven political polarization makes economy more distorted. To this purpose, the degree of politico-economic output distortion is measured by comparing the total output under the politically chosen policies with the total output under socially optimal policies that a benevolent social planner would choose. It turns out that capital-biased technological progress does not worsen politico-economic output distortion, while it exacerbates political polarization. Neither biased nor unbiased technological progress affects politico-economic output distortion.

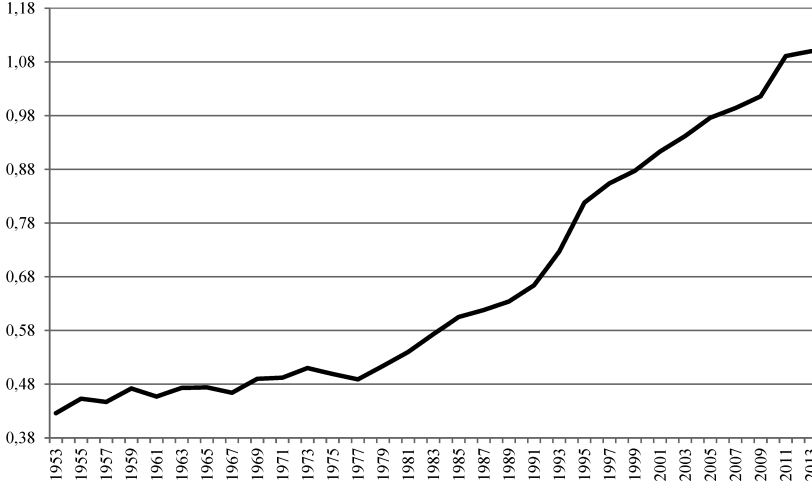
The remainder of this paper unfolds as follows. Section 2 delineates our model and types of technological progress. Section 3 characterizes competitive market equilibrium allocation. Section 4 identifies each voter’s ideal taxation policies, from which the degree of political polarization is determined. Section 5 and 6 analyze the effects of each type of technological progress on political polarization and on politico-economic distortion. Section 7 quantifies the effects using US data. Section 8 concludes.

Figure 1
TRENDS OF RELATIVE CAPITAL PRODUCTIVITY OF THE US



Note: With standard Cobb-Douglas production function, capital income share is output elasticity with respect to capital input and represents relative contribution to output; hence, it indicates relative capital productivity. The income share data is from the US Bureau of Labor Statistics.

Figure 2
TREND OF POLITICAL POLARIZATION OF THE US



Note: The above figure reports difference between the mean DW-NOMINATE scores of Democratic Party and Republican Party members in the House of Representatives, as DW-NOMINATE score measures political ideology with the US Congress roll-call voting data. For details of how the DW-NOMINATE score is calculated, refer to Poole and Rosenthal (1997), McCarty *et al.* (1997) and Poole (2005).

2. Economic Environment

2.1. Individual Voters

Consider an economy populated by individual voters who are born with different earning abilities and are indexed by i . Equal amount of capital endowment is given to each of individual voters when they are born. The size of the population is one. This economy is in its steady state; thus, time subscripts are dropped. For any given i , the lifetime utility² of individual voter i is

$$(1 - \beta) \sum_{t=1}^{\infty} \beta^{t-1} \left[\log(c_i) - \frac{l_i^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} + \chi \log(G) \right] \quad (1)$$

where $\beta \in (0,1)$ is time preference; $c_i > 0$ and $l_i \in [0,1]$ are private goods consumption and labor supply, respectively, of individual voter i ; $\eta > 0$ is Frisch elasticity of labor supply; G is public goods provided by the government of this economy; $\chi > 0$ is preference for public goods. In maximizing the utility function of (1), individual voter i meets the following inter-temporal budget constraint:

$$w\theta_i l_i - T_L(w\theta_i l_i) + (k_i + rk_i - T_K(rk_i)) \geq c_i + k'_i \quad (2)$$

where w and r are market wage and interest rates, respectively; $T_L(w\theta_i l_i)$ and $T_K(rk_i)$ are labor and capital income taxes, respectively, imposed on individual voter i ; and, k_i and k'_i are capital investment (i. e., savings) made by individual voter i in the previous period and in the current period, respectively. Moreover, earning ability of individual voter i is

$$\theta_i = \exp(\pi_i + \varepsilon_i) \quad (3)$$

where π_i and ε_i are given at birth and remain the same for the lifetime. The support of the distribution of θ_i is denoted by Θ . π_i is distributed according to *Exponential* $\left(\frac{1}{\sigma_\pi}\right)$, and ε_i is distributed according to *Normal* $\left(-\frac{\sigma_\varepsilon}{2}, \sigma_\varepsilon\right)$. Thus, $\exp(\pi_i)$ follows a Pareto distribution, and $\exp(\varepsilon_i)$ follows a Lognormal distribution, generating a thick upper tail in the earnings distribution to closely resemble actual income distributions (e. g., Armour *et al.*, 2016). Although most of theoretical studies about income distribution adopts Lognormal distribution, it is criticized that the upper tail of Lognormal distribution is too thin to properly reflect actual distributions of high incomes. At the same time, Pareto distribution cannot properly reflect actual distributions of middle and low income, although its upper tail is suitable for resembling actual distribution of high incomes. To overcome these limitations, Heathcote *et al.* (2014 and 2017) combined the two distributions by representing earning ability with two variables in the equation (3). To closely reflect the entire part of actual income distributions, this study also adopts their method of parameterizing earning ability with combining Lognormal and Pareto distributions.

With their earning ability known publicly, in each period, individual voters choose their own labor supply and private goods consumption from maximizing their own utility for the remaining lifetime. For any given $\theta_i \in \Theta$, the maximization problem that individual voter i solves in each period is stated, in a recursive way, as

$$v(k_i; \theta_i) = v_i(k_i) = \max\left\{(1 - \beta)[\log(c_i) - \frac{l_i^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} + \chi \log(G)] + \beta E[v_i(k'_i)] \text{ s.t. (2)}\right\}. \quad (4)$$

2.2. Firms

In this economy, a representative firm produces output Y that can be used for private goods and public goods consumption. That is, with $1 > \alpha > 0$,

$$Y = F(K, L) = z_T(z_K K)^\alpha (z_L L)^{1-\alpha} \quad (5)$$

where K is aggregate capital and L is aggregate labor in efficiency unit. In each period, the representative firm chooses its inputs of capital and labor by solving the profit maximization problem of $\max z_T(z_K K)^\alpha (z_L L)^{1-\alpha} - rK - wL$. The production function (5) is standard Cobb-Douglas function. Even though the elasticity of substitution of Cobb-Douglas produc-

tion function is set as one, most of studies in public economics and macroeconomics adopt Cobb-Douglas function. Moreover, the empirical study of León-Ledesma and Satchi (2019) rigorously showed that the elasticity of substitution is one in the long run that is pertinent to our analysis describing a steady state of the economy.

2.3. Technology

Unlike decisions on the inputs, technology breakthroughs or innovations are not easily controllable or choosable; hence, we describe technological progress as an increase in the given parameter of production function (5). That is, an increase in the value of each of the parameters of z_T , z_K , z_L and α represents a form of technological progress.³ An increase in z_T is technological progress that improves total factor productivity; an increase in z_K is capital-augmenting technological progress; and, an increase in z_L is labor-augmenting technological progress. Obviously, these three forms of technological progress are different from each other. For example, as $1 > \alpha > 0$, an increase in z_K by 1.5 yields strictly different amounts of increases in marginal factor productivity and total output than an increase in z_T or z_L by 1.5 does. In spite of the difference, increases in z_T , z_K or z_L share the following common feature: They preserve the relative factor productivity. Since the relative capital pro-

ductivity is $\frac{F_K(K, L) \frac{K}{\bar{Y}}}{F_L(K, L) \frac{L}{\bar{Y}}} = \frac{\alpha}{1 - \alpha}$ and its reciprocal ($\frac{1 - \alpha}{\alpha}$) is the relative labor productivity,

an increase in the value of any one of the three technology parameters of z_T , z_K , and z_L does not favor the relative capital productivity or the relative labor productivity. Therefore, based on this common feature, the three forms of technological progress –total factor productivity improvement, capital-augmenting technological progress and labor-augmenting technological progress– are classified into one type of technological progress: “unbiased technological progress”.

By contrast to an increase in z_T , z_K or z_L , an increase or a decrease in the technology parameter α does not preserve the relative factor productivity but favors the relative productivity of capital or labor. Thus, a change in the value of α represents another type of technological progress: “biased technological progress.” An increase in α is capital-biased technological progress, as it raises the relative capital productivity and lowers the relative labor productivity. A decrease in α is labor-biased technological progress, as it raises the relative labor productivity and lowers the relative capital productivity. This implies that labor-biased technological progress is equivalent to capital-biased technological *regress*. In fact, the effect of labor-biased technological progress (a decrease in α) is simply opposite to the effect of capital-biased technological progress (an increase in α). In this light, for analyzing the effect of biased technological progress, it is sufficient to examine the effect of capital-biased technological progress, based on which it is trivial to identify the effect of labor-biased technological progress. Moreover, capital-biased technological progress is more relevant than labor-biased technological progress, due to the recent waves of automation or robotization which obviously raise the relative capital productivity. Thus, for an efficient and

relevant analysis, here, we just examine the effect of capital-biased technological progress (i. e., an increase in α) instead of those of both labor-biased and capital-biased technological progresses. Fundamentally, notice that α is a technology parameter as it indicates output elasticity with respect to capital input, independent of factor markets or firm's input decisions. It is *only* after factor markets clear when α can also indicate the capital income share (i. e., the ratio of the total capital income to the total output).

The four technology parameters of z_T, z_K, z_L and α , whose changes represent a biased or unbiased technological progress, are not time-variant and shall be later allowed to increase for identifying the effect of each type of technological progress.⁴ Because the US Federal Reserve Bank estimates total factor productivity with assuming that $z_K = 1$ and $z_L = 1$, its estimate outcome of total factor productivity is translated into $z_T z_K^\alpha z_L^{1-\alpha}$ in light of our model that allows z_K and z_L to take values other than 1. On the other hand, regardless of whether assumption of $z_K = 1$ and $z_L = 1$ is imposed or not, capital income share (data of the grey line in Figure 1) estimates the value of α . Hence, in light of (5), the upward trend of the grey line of Figure 1 evidences capital-biased technological progress. Notably, it is clear that total factor productivity (labor-augmenting or capital-augmenting technology productivity) estimated will be showing aspects of technological progress that cannot be captured by estimated α . In reality, biased technological progress can occur at the same time when unbiased technological progress proceeds, although both types of technological progress are distinctively different from each other. Our model can help disentangle them and identify their respective effects.

2.4. Government

The government of this economy collects capital and labor income taxes to finance public goods provision, in each period, meeting the following budget constraint.

$$\int_{\Theta} \{T_L(w\theta l_i(\theta)) + T_K(rk_i(\theta))\} dF_\theta = G = gY \quad (6)$$

where $g \in (0,1)$ is the fraction of the total output earmarked for public goods provision. Moreover, capital income tax is linear. That is,

$$T_K(rk_i) = \tau_K rk_i \quad (7)$$

with $\tau_K \in [0,1]$ being capital income tax rate. At the same time, labor income tax is nonlinear taking a form of the following function:

$$T_L(y_i) = y_i - \rho_L (y_i)^{1-\mu_L} \quad (8)$$

where $y_i = w\theta l_i$ (pre-tax labor income). According to (8), post-tax labor income is $\rho_L (y_i)^{1-\mu_L}$. If $1 - \mu_L \leq 0$ or $\rho_L \leq 0$, more labor supply does not bring more disposable income causing individuals not to supply labor. For inducing individual voters to work and earn a positive amount of taxable income,

$$1 - \mu_L > 0 \quad \text{and} \quad \rho_L > 0. \quad (9)$$

Markedly, μ_L represents the progressivity of labor income tax rate schedule. In particular, labor income tax is progressive if $\mu_L > 0$ and regressive if $\mu_L < 0$, because marginal labor income tax rate $T_L'(y_i) = 1 - \rho_L(1 - \mu_L)(y_i)^{-\mu_L}$ increases with pre-tax labor income if $\frac{dT_L'(y_i)}{dy_i} = \rho_L\mu_L(1 - \mu_L)(y_i)^{-\mu_L-1} > 0$ and decreases otherwise. If $\mu_L = 0$, labor income tax is linear, as capital income tax of (7). Being differentiated from the most studies of Ramsey taxation literature, which assumed labor income tax to be linear, this paper relaxes this assumption so that labor income tax can be nonlinear. While individual voters are endowed with equal amount of capital, they have unequal abilities to earn labor incomes, which is consistent with equal rate of tax on capital incomes (linear capital income tax) and unequal rates of tax on labor incomes (nonlinear labor income tax). Corresponding to the average marginal capital income tax rate that is equal to τ_K , the average marginal labor income tax rate τ_L is defined as

$$\tau_L = \int_{\Theta} \left\{ T_L'(y_i(\theta)) \left(\frac{y_i(\theta)}{Y} \right) \right\} dF_{\theta}. \quad (10)$$

With any given μ_L that determines the slope of labor income tax rate schedule, according to (8), ρ_L determines the level of the average marginal labor income tax rate.

As a matter of fact, the labor income tax function of (8) has been adopted by various studies such as Feldstein (1969), Persson (1983), Benabou (2000 and 2002), Corneo (2002), Heathcote *et al.* (2014 and 2017), Guerreiro *et al.* (2018), and the like. Heathcote *et al.* (2017) showed that the tax function (8) is remarkably well fitted to the US data. Moreover, the linear capital income tax of (7) has been most widely adopted in Ramsey taxation literature and macroeconomics literature. Being consistent with these vast literatures, the linearity of (7) is necessary for tractable analysis.

Without specifying the functions of utility, production and taxes, it is not feasible to identify how technological progress affects political polarization, although it does not attain the highest level of generality. Nonetheless, the utility function of (1), the production function of (5) and the tax functions of (7) and (8) still can yield policy-relevant implications, because these functions themselves are supported by empirical data. The logarithm consumption utility of (1) has been widely adopted by a great number of studies, as it is shown to be consistent with empirical findings. As pointed out by Kimball and Shapiro (2008), numerous empirical studies have found that estimated income effect and substitution effect of wage on labor supply are of approximately the same size to cancel each other out, which gives empirical supports to logarithm consumption utility.

By the majority-rule voting, voters democratically decide the set of the government policies of τ_K , τ_L , μ_L , ρ_L and g in each period. Each individual has one vote. The set of the government policies that beats any other alternative set of the government policies in the majority-rule voting is selected as politically optimal set of policies. Notice that according to

(6), (7), (8) and (10), once τ_K , τ_L and μ_L are decided, g and ρ_L are automatically determined. Thus, with the balanced fiscal budget, voting essentially takes place only on the set of τ_K , τ_L and μ_L . First, each voter casts their own vote to a set of the government policies (τ_K , τ_L and μ_L). Then, after the government policies are decided by the majority-rule voting (as a politically optimal set of policies), each individual voter supplies labor as well as consumes private goods and public goods.

2.5. Competitive General Equilibrium

As each individual voter makes their own voting decision over any two sets of candidate policies of the government, he prefers the set of the government policies that gives higher level of utility to him himself, without considering others' utility. In evaluating a set of candidate policies of the government to make a voting decision, level of each voter's utility depends on prices of labor and capital as well as on total output which are determined by competitive markets (not by government or by a voter) under the set of the policies. Because the production function (5) and how political decisions on policies will be made are publicly known, each individual voter can calculate the prices and total output for any given set of candidate policies. In contrast to the government policies, allocation of labor and capital that determines total output as well as their prices are not politically decided by voting. Rather, both total output and prices of its inputs (labor and capital) are determined by markets with reaching a competitive general equilibrium of this economy. Hence, when each individual voter evaluates any set of τ_K , τ_L , μ_L , ρ_L and g , he needs to take competitive-equilibrium prices and allocation into account. In other words, for each voter to properly assess a given set of the government policies, prices and aggregate allocation that are necessary in assessing level of each voter's utility from the given set of the government policies should be supported as a competitive general equilibrium of this economy.

With the government policies given, a competitive general equilibrium of this economy is defined as a set of allocation decision rules $\{c_i(\theta_i), l_i(\theta_i), k_i(\theta_i)\}_{\theta_i \in \Theta}$ and prices of labor and capital (w and r) which satisfies the following three conditions in each period:

- (i) With the government policies and prices given, each individual voter maximizes their own utility meeting their own budget constraint.
- (ii) The representative firm maximizes its profit with the factor markets being cleared as

$$K = \int_{\Theta} k_i(\theta) dF_{\theta}, \quad (11)$$

$$L = \int_{\Theta} \theta l_i(\theta) dF_{\theta}. \quad (12)$$

- (iii) The government's budget constraint of (6) is met.

Once the budget balances of all individual voters and the government are met with the factor markets being cleared, due to Walras' law, the aggregate resource constraint of this economy is automatically met, clearing the goods market as well. Moreover, as this economy is in its steady state, the competitive-equilibrium amount of k_i stays the same over time; that is, $k_i = k'_i > 0$, for any given $\theta_i \in \Theta$.

For each voter to assess level of their own utility from a given set of the government policies in this market economy, the above conditions (i), (ii) and (iii) should be met to ensure that prices and aggregate allocation (necessary elements for utility assessment) be supported as a competitive general equilibrium. Based on such assessments of the government policies with competitive markets deciding prices and allocation, each individual voter not only can make their own voting decision but also can identify what is their own ideal set of the government policies that maximizes their own utility. Then, the degree of political polarization is determined from difference between voters' ideal policies identified. To efficiently analyze how technological progress affects political polarization, we translate competitive-equilibrium allocation in terms of the government policy variables in the next section. By restating competitive-equilibrium allocation in terms of the government policy variables and then plugging the restated competitive-equilibrium allocation directly into the utility function of (1), we obviate the need for Lagrange multiplier(s)⁵ of the conditions (i), (ii) and (iii) in identifying each voter's ideal policies.

3. Competitive Market Equilibrium Allocation in Terms of Policy Variables

To obtain competitive-equilibrium allocation that meets the above three conditions (i), (ii) and (iii), we begin with characterizing the optimal allocation decision rules of private consumption, labor supply and savings of each individual voter. To this end, for any given $\theta_i \in \Theta$, let us denote the ratio of post-tax labor income to post-tax capital income of individual voter i by

$$s_i(\theta_i) = \frac{\rho_L(w\theta_i l_i)^{1-\mu_L}}{r k_i(1-\tau_K) + k_i - k'_i}. \quad (13)$$

The method of using this auxiliary variable is also used by other studies like Benabou (2002) as it streamlines the analysis. As the budget constraint of (2) binds at a competitive general equilibrium, plugging (7) and (8) in the balanced budget constraint

yields $c_i(\theta_i) = \left(1 + \frac{1}{s_i}\right) \rho_L(w\theta_i l_i)^{1-\mu_L}$. Hence, the optimal private consumption of individual

voter i can be defined completely by his optimal labor supply and his optimal ratio of post-tax labor income to post-tax capital income. Thus, re-stating the value function as

$$(1-\beta) \left[\log \left(\left(1 + \frac{1}{s_i}\right) \rho_L(w\theta_i l_i)^{1-\mu_L} \right) - \frac{l_i^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \chi \log(G) \right] + \beta E[v_i(k'_i)]$$

enables us to characterize optimal allocation of individual voter i as follows. For any given $\theta_i \in \Theta$, we use

the above value function restated for FOC with respect to labor supply so that the optimal labor supply of individual voter i is defined by

$$(1 - \beta) \frac{(1 - \mu_L)}{l_i} - (1 - \beta) l_i^{\frac{1}{\eta}} + \beta E \left[\frac{dv(k'_i)}{dk'_i} \frac{dk'_i}{dl_i} \right] = 0. \quad (14)$$

Likewise, using the restated value function, the optimal ratio of post-tax labor income to post-tax capital income of individual voter i is defined by

$$(1 - \beta) \frac{1}{\left(1 + \frac{1}{s_i}\right)} (-1) \left(\frac{1}{s_i}\right)^2 + \beta E \left[\frac{dv(k'_i)}{dk'_i} \frac{dk'_i}{ds_i} \right] = 0. \quad (15)$$

By the same token, plugging (7) and (8) in the balanced budget constraint also yields $c_i = (1 + s_i)(rk_i(1 - \tau_K) + k_i - k')$ so that we can obtain FOC with respect to k' . From this, the optimal inter-temporal allocation of individual voter i is defined by the following Euler equation.

$$\frac{1}{rk_i(1 - \tau_K) + k_i - k'_i} = \beta [1 + r(1 - \tau_K)] E \left[\frac{1}{rk'_i(1 - \tau_K) + k'_i - k''_i} \right] \quad (16)$$

where k''_i is capital investment made by individual voter i in the next period; and, at the steady state, $k_i = k'_i = k''_i > 0$. Based on the optimality conditions of (14), (15) and (16), we identify each individual voter's optimal allocation at a competitive general equilibrium of this economy that is in its steady state.

Lemma 1. At a stationary competitive general equilibrium, with the government policies and prices given, for any given $\theta_i \in \Theta$, the optimal labor supply, private consumption, and the optimal ratio of post-tax labor income to post-tax capital income of individual voter i are as follows:

$$l_i(\theta_i) = \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}}, \quad (17)$$

$$c_i(\theta_i) = (\theta_i)^{1-\mu_L} \frac{\rho_L(w)^{1-\mu_L} \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}(1-\mu_L)}}{1 - \beta}, \quad (18)$$

$$s_i(\theta_i) = \frac{1 - \beta}{\beta}. \quad (19)$$

For proof, see Appendix A1.

Basically, *Lemma 1* characterizes the condition (i) for competitive-equilibrium allocation. (17) shows that more progressive labor income tax reduces individual voters' labor supply. With the logarithm utility of consumption, income and substitution effects on labor supply of the effective wage rate ($w\theta_i$) cancel out each other.⁶ As a result, the optimal

labor supply and the optimal ratio of post-tax labor income to post-tax capital income of individual voters are independent of earning ability, as shown in (17) and (19). However, individual voters' private goods consumption depends positively on earning ability as well as post-tax labor income, as appears in (18). Furthermore, as $s_i(\theta_i) > 0$ for any given $\theta_i \in \Theta$, individual voters' private goods consumption also depends positively on post-tax capital income. Although all individuals are born with equal amount of capital endowment, due to earning-ability inequality among them, individuals of higher ability earn more post-tax capital incomes because they are paid higher wage. On the other hand, as (19) indicates, $s_i(\theta_i)$ is independent of earning ability; hence, the ratio of the contribution of labor income for private consumption to that of capital income is the same for individuals of different earning abilities.

By aggregating individual voters' optimal allocation (*Lemma 1*), we obtain total supplies of labor and capital, from which we calculate marginal products of aggregate labor and capital. Then, we equate the marginal products of aggregate labor and capital with market-clearing wage and interest rates, respectively, to satisfy the competitive-equilibrium condition (ii). To meet the competitive-equilibrium condition (iii), the government's budget constraint of (6) is restated in terms of the obtained optimal allocation of individuals, market-clearing wage and interest rates. From this, we identify ρ_L that meets the competitive-equilibrium condition (iii). Because of (19) and $k_i = k'_i$ for any given $\theta_i \in \Theta$ at a stationary competitive general equilibrium, ρ_L is uniquely determined once μ_L and τ_K are decided with the government's budget being balanced. As a consequence, each voter only needs to vote on which set of μ_L and τ_K as all the remaining government policies are automatically determined by reaching a stationary competitive general equilibrium. Moreover, we can define the steady-state aggregate competitive-equilibrium allocation that meets the conditions (i) (ii) and (iii), in terms of μ_L and τ_K , as below.

Lemma 2. At a stationary competitive general equilibrium, the aggregate labor, capital and output are as follows:

$$L = \frac{1}{(1 - \sigma_\pi)} \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}}, \quad (20)$$

$$K = z_T^{\frac{1}{(1-\alpha)}} z_K^{\frac{\alpha}{(1-\alpha)}} z_L \left(\frac{\beta}{1-\beta} \right)^{\frac{1}{(1-\alpha)}} \alpha^{\frac{1}{(1-\alpha)}} (1 - \tau_K)^{\frac{1}{(1-\alpha)}} \frac{1}{(1 - \sigma_\pi)} \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}}, \quad (21)$$

$$Y = z_T^{\frac{1}{(1-\alpha)}} z_K^{\frac{\alpha}{(1-\alpha)}} z_L \left(\frac{\beta}{1-\beta} \right)^{\frac{\alpha}{(1-\alpha)}} \alpha^{\frac{\alpha}{(1-\alpha)}} (1 - \tau_K)^{\frac{\alpha}{(1-\alpha)}} \frac{1}{(1 - \sigma_\pi)} \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}}. \quad (22)$$

At a stationary competitive general equilibrium, the fraction of the aggregate output used for public goods provision is

$$g = 1 - \frac{\alpha}{\beta} (1 - \tau_K). \quad (23)$$

For proof, see Appendix A2.

As shown by (20) and (21), labor income tax progressivity affects both inputs of aggregate capital and aggregate labor supply, because individual voters supply capital from their post-tax labor incomes. In turn, (22) and (23) show that an increase in the degree of labor income tax progressivity or in the capital income tax rate reduces aggregate output by lowering individual voters' labor-supply and savings incentives, while the increase leads to more public goods provision. Thus, increasing utility by public goods consumption comes at the cost of the efficiency loss of reducing total output. The competitive-equilibrium provision of public goods (23) is defined from the fiscal budget balance condition of (6). With (19) of *Lemma 1*, the competitive-equilibrium provision of public goods also can be equivalently stated in terms of μ_L and ρ_L , which is much more complicated and less informative than (23).

Incorporating the competitive-equilibrium conditions (i), (ii) and (iii), which are delineated by *Lemma 1 and 2* in terms of policy variables, we obtain each individual voter's preference for government policies, U_i , as follows: for any given $\theta_i \in \Theta$,

$$\begin{aligned}
 U_i = (1 + \chi) & \left\{ \frac{1}{(1 - \alpha)} \log(z_T) + \log(z_L) + \frac{\alpha}{(1 - \alpha)} \log(z_K) \right\} + (1 - \mu_L)\theta_i + \\
 & (\chi + 1) \frac{\eta}{\eta + 1} \log\{(1 - \mu_L)(1 - \beta)\} - \frac{\eta}{\eta + 1} \{(1 - \mu_L)(1 - \beta)\} + \\
 & \frac{(\alpha\chi + 1)}{(1 - \alpha)} \log(1 - \tau_K) + \chi \log\left(1 - \frac{\alpha}{\beta}(1 - \tau_K)\right) + \log(1 - \sigma_\pi(1 - \mu_L)) + \\
 & \frac{\sigma_\varepsilon \mu_L (1 - \mu_L)}{2} + \Gamma
 \end{aligned} \tag{24}$$

where $\Gamma = \frac{\alpha(\chi + 1)}{(1 - \alpha)} \log\left(\frac{\beta}{1 - \beta}\right) - \log(\beta) + \frac{\alpha\chi + 1}{(1 - \alpha)} \log(\alpha) - (\chi + 1) \log(1 - \sigma_\pi)$. The first three terms of the indirect utility function of (24) show that unbiased technological progress (an increase in z_T , z_K or z_L) always increases each voter's utility, which is independent of the government policies. In contrast, whether and how much capital-biased technological progress (an increase in α) increases each voter's utility is not independent of the government policies. More importantly, the fourth term of (24) differs across voters of different earning abilities, generating gaps in voters' ideal policies for this economy. Because voters of different earning abilities face different labor income tax rates, the fourth term that creates ideological difference between voters depends only on μ_L . On the other hand, as public goods are equally provided to all voters regardless of their earning ability, utility gain from public goods consumption that is the eighth term of (24) is the same for individuals of different earning abilities and depends on τ_K .

Now, based on the indirect utility function of (24) that embeds the competitive-equilibrium conditions (i), (ii) and (iii), each voter can evaluate any given sets of the government policies to make their own voting decision as well as to find their own ideal policies. In the

evaluation, allocation under any given set of the government policies is implemented voluntarily by individuals and the firm as a competitive general equilibrium. By assessing candidate sets of the policies based on (24), voters cast their own vote over the candidate sets of the government policies. Thus, the majority-rule voting that selects the government policies is multi-dimensional. Nonetheless, because the indirect utility function of (24) of each voter satisfies intermediate preference, we can apply the Median Voter Theorem to identify a politically optimal set of the government policies (Persson and Tabellini, 2002). Thus, the ideal policies of the median voter (i. e., the voter whose earning ability is the median of θ_i) will be the politically optimal set of the government policies.

4. Ideal Policy of Each Voter and Political Polarization

While only one set of the government policies is selected as the politically optimal set of policies by the majority-rule voting, the politically chosen set of the policies does not necessarily maximize the utility of each individual voter. Rather, individual voters have different ideal set of government policies that maximizes their own utility. That is, voters have disagreement over which set of the government policies should be taken. Because such disagreement gives birth to political polarization, measuring the degree of political polarization is based on the distance of different voters' ideal policies. Thus, to pave the way for analyzing the effect of technological progress on political polarization, we first need to identify each voter's ideal set of government policies.

To this purpose, it is sufficient to identify each voter's own ideal values of τ_K , τ_L and μ_L . As noted above, these three policy variables automatically determine the remaining two policy variables of ρ_L and g at a competitive general equilibrium that balances the government budget. Thus, a gap in voters' ideal values of the latter two policy variables (ρ_L and g) is redundant in reflecting voters' disagreement over ideal policies. To avoid over-estimation of the degree of political polarization from voters' policy preferences, we identify the degree of political polarization with the distance between left-wing and right-wing voters' own ideal values of τ_K , τ_L and μ_L .

For any given $\theta_i \in \Theta$, we can obtain voter i 's ideal set of the government policies of τ_K^i , τ_L^i and μ_L^i by solving the problem of unconstrained maximization of (24). Because all the competitive-equilibrium conditions (i), (ii) and (iii) are already embedded into the utility function of (24), prices and allocation that are factored in assessing voter i 's utility are supported as a competitive general equilibrium.

Proposition 1. For any given $\theta_i \in \Theta$, the set of the government policies (τ_K^i , τ_L^i and μ_L^i) ideal for a voter who is born with the earning ability of θ_i is defined as follows.

$$\tau_k^i = \begin{cases} 1 - \frac{\beta}{\alpha} \left(\frac{\alpha\chi + 1}{\chi + 1} \right) & \text{if } \alpha(\chi + 1) > \beta(\alpha\chi + 1), \\ 0 & \text{if } \alpha(\chi + 1) \leq \beta(\alpha\chi + 1) \end{cases}, \quad (25)$$

$$\tau_L^i = (1 - \alpha) - (1 - \mu_L^i) \left(\frac{\alpha\chi + 1}{\chi + 1} \right), \quad (26)$$

$$\frac{\sigma_\pi}{1 - \sigma_\pi(1 - \mu_L^i)} + \sigma_\varepsilon \left(\frac{1}{2} - \mu_L^i \right) + \frac{\eta}{\eta + 1} (1 - \beta) - \theta_i = \frac{\eta}{\eta + 1} \frac{(1 + \chi)}{(1 - \mu_L^i)}. \quad (27)$$

For proof, see Appendix A3.

Basically, voters finance their public goods consumption with income tax revenue. While non-distortionary lump-sum tax is not available, the policy instrument of income taxation is distortionary. Income taxation creates input-price distortions for individual voters who provide the inputs of capital and labor, which in turn reduces total output for voters' private and public goods consumption. While income taxation incurs this efficiency loss, public goods provision from income taxation offers utility gains to voters. *Proposition 1* shows that this trade off underlies each voter's ideal set of the government policies. From (25) and (26) of *Proposition 1*, it is straightforward that each voter's ideal rates of capital and labor income tax increase with their preference for public goods (χ).⁷

Because individual voters of different earning abilities face the same capital income tax rate and the same market price of capital, both high-ability voters and low-ability voters suffer same degree of capital-price distortion. At the same time, as shown by (24) above, both high-ability voters and low-ability voters enjoy the same level of utility gain from public goods consumption. Therefore, ideal rate of capital income tax is the same for high-ability voters and low-ability voters, as appears in (25). That is, voters have no disagreement over the ideal rate of capital income tax.

On the other hand, however, individual voters of different earning abilities face different labor income tax rates and different market prices of labor ($w\theta_i$). As a result, high-ability voters and low-ability voters suffer different degrees of labor-price distortion, although all of them get the same utility gain from public goods consumption. Hence, as shown by (26), high-ability voters and low-ability voters now have disagreement over the ideal level of average marginal labor income tax rate. The average marginal labor income tax rate represents the *entire* schedule of labor income tax rates, which depends on μ_L and ρ_L . For any given μ_L , (6), (8) and (10) imply that deciding τ_L is equivalent to deciding ρ_L and can reflect the contribution of labor income tax for providing public goods that are produced by the firm. Consequently, as appears in each term of (26), for any given $\theta_i \in \Theta$, voter i 's ideal rate of average marginal labor income tax depends on labor income share $(1 - \alpha)$ and μ_L^i . Notice from (8) that μ_L governs the link from individual-specific earning ability to individual-specific labor income tax rate while fiscal budget balance and aggregate contribution of labor income tax for production of public goods lead to τ_L (or ρ_L equivalently). This implies that voters' disagreement over the ideal rate of average marginal labor income tax originates from their disagreement over the ideal slope of labor income tax rate schedule.

The right-hand side of (27) is the marginal benefit from an increase in the slope of labor income tax rate schedule that increases revenue for public goods provision. On the other hand, the

left-hand side of (27) is the marginal cost from an increase in the slope of labor income tax rate schedule that reduces total output for private goods consumption. As shown in (17) of *Lemma 1*, an increase in the slope of labor income tax rate schedule (labor income tax progressivity) lowers labor supply of all individuals of different earning abilities to lower total output so that aggregating this labor-supply disincentive involves the two parameters σ_π and σ_ϵ of the earning-ability distribution. Since *Lemma 1* implies that voters of higher earning ability enjoy higher level of private goods consumption, the marginal cost is lower for voters of higher earning ability, as shown by the fourth term of the left-hand side of (27). After all, each voter's ideal degree of labor income tax progressivity is set to equate the marginal benefit with the marginal cost that is different across voters. As such, voters are divided on the ideal slope of labor income tax rate schedule.

Now, based on the ideal taxation policies of each voter obtained in *Proposition 1*, we can identify the degree of political polarization (denoted by ϕ) with the distance between the ideal taxation policies of a left-wing voter (a voter whose earning ability is lower than $med[\theta_i]$ where $med[\theta_i]$ is the median of θ_i) and of a right-wing voter (a voter whose earning ability is higher than $med[\theta_i]$). That is, for $\theta_{right} > med[\theta_i] > \theta_{left}$,

$$\phi = |\tau_K^{left} - \tau_K^{right}| + |\tau_L^{left} - \tau_L^{right}| + |\mu_L^{left} - \mu_L^{right}| \quad (28)$$

where τ_K^{left} , τ_L^{left} and μ_L^{left} are the ideal values of capital income tax rate, average marginal labor income tax rate and labor income tax progressivity, respectively, of a left-wing voter; and, τ_K^{right} , τ_L^{right} and μ_L^{right} are those of a right-wing voter. As ideal policies of a voter represent ideology of the voter, the degree of political polarization of (28) reflects ideological distance between left-wing and right-wing voters. Moreover, as *Lemma 1* implies that pre-tax income increases with earning ability, a right-wing voter is richer than a left-wing voter, and the median voter earns the median level of pre-tax income. Markedly, differentiated from the existing studies on political polarization (e.g., Alesina and Tabellini, 1990; Azzimonti and Talbert, 2014), in this study, political polarization is not exogenously given as a parameter but *endogenously* derived from individual voters' ideal taxation policies that can be affected by production technology advancements. Moreover, this study does not impose the assumption that individuals are born as worker vs. non-working capitalist (or liberal vs. conservative).

Notice that $\phi > 0$ even without introducing any technological progress. Although (25) of *Proposition 1* implies that $\tau_K^i = \tau_K^j$ for any i and j , there exists disagreement over the other government policies before introducing a change in any one of the four technology parameters (z_T , z_K , z_L and α). To show that $\phi > 0$ in this baseline economy, we present the following corollary to *Proposition 1* that describes how left-wing and right-wing voters' ideal policies are strictly different from each other's.

Proposition 2. For any given i and j , whenever $\theta_i > \theta_j$, $\mu_L^i < \mu_L^j$ and $\tau_L^i < \tau_L^j$.

For proof, see Appendix A4.

Because an increase in the degree of labor income tax progressivity stiffens the slope of labor income tax rate schedule, it raises tax rate more for voters of higher earning ability.

As $\theta_{right} > \theta_{left}$, high-ability voters (right-wing voters) strictly prefer lower degree of tax progressivity than low-ability voters (left-wing voters), begetting political polarization of this economy. For any given amount of labor supply, an increase in marginal labor income tax rate takes larger amount of tax away from high-ability voters than from low-ability voters. Hence, right-wing voters strictly prefer lower level of average marginal labor income tax rate than left-wing voters, which also generates gap between left-wing and right-wing voters' ideal policies. Notably, *Proposition 2* and (3) suggest that political polarization originates from differences in voters' ability to earn labor incomes. Therefore, even without introducing any technological progress, left-wing and right-wing voters are divided over what taxation policies should be chosen for this economy.

As production technology determines voters' pre-tax returns on labor and capital, technological progress can change gap of ideal taxation policies between left-wing and right-wing voters by altering their ideal policies. Based on voters' ideal policies characterized in this section, we analyze how technological progress affects political polarization in the next section.

5. Effect of Technological Progress on Political Polarization

So far, with production technology *fixed*, we have obtained voters' ideal policies and identified the degree of political polarization. Now, this section introduces an increase in each of the four technology parameters (z_T , z_K , z_L and α) to examine how technological progress affects political polarization. As described in Section 2, technological progress is classified into either unbiased technological progress (an increase in z_T , z_K or z_L) or biased technological progress (an increase or a decrease in α), depending on whether it preserves the relative factor productivity or not. To begin, we introduce an increase in z_T , z_K and z_L .

Proposition 3. Unbiased technological progress does not affect the degree of political polarization. Moreover, if technological progress is unbiased, it does not change any voter's ideal levels of capital income tax rate and average marginal labor income tax rate or any voter's ideal degree of labor income tax progressivity.

For proof, see Appendix A5.

As shown by (27) of *Proposition 1*, individual-specific earning ability and the earning-ability distribution, which is determined by σ_ε and σ_π , shape individual voter's ideal slope of labor income tax schedule. Because unbiased technological progress does not change any voter's earning ability or the earning-ability distribution, it does not change any voter's ideal degree of labor income tax progressivity. More importantly, by preserving the relative factor productivity, unbiased technological progress does not change how much each input of capital or labor is relatively more necessary to yield one unit of output, which preserves the ratio of pre-tax prices of capital and labor and thus input-price distortions. It is straightforward from (5) that unbiased technological progress does not alter the price elasticities of capital and labor. Consequently, voters' disincentives for supplying capital and labor traded off for public-goods benefit remain unaltered when technological progress is unbiased. Because this

trade off steers voters' ideal levels of tax rates on capital and labor incomes, as shown by (25) and (26) of *Proposition 1*, unbiased technological progress does not affect any voter's ideal levels of capital income tax rate and average marginal labor income tax rate. Thus, none of labor-augmenting technological progress, capital-augmenting technological progress and total factor productivity improvement affects τ_K^i , τ_L^i and μ_L^i for any given $\theta_i \in \Theta$. As it does not change any voter's ideal policies, unbiased technological progress does not affect political polarization.

Having examined the effect of unbiased technological progress on political polarization, we now analyze the effect of biased technological progress (an increase or a decrease in α). Because labor-biased technological progress (a decrease in α) is equal to capital-biased technological regress and exerts the opposite effect of capital-biased technological progress (an increase in α), we only examine the effect of capital-biased technological progress on political polarization, which is efficient and sufficient for analyzing the effect of biased technological progress. While an increase in z_T , z_K or z_L always increases total output, an increase in α does not always do so. If a technological innovation results in decreasing total output, it is not a progress but a regress. To rule this out, we focus on the relevant case where an increase in α increases total output by meeting the following condition:

$$\frac{dF(K, L)}{d\alpha} = z_T(z_K K)^\alpha (z_L L)^{(1-\alpha)} \ln\left(\frac{z_K K}{z_L L}\right) > 0. \quad (29)$$

In fact, the condition of (29) is not restrictive, because the theoretical findings of this paper hold regardless of whether (29) is met or not.

Proposition 4. Capital-biased technological progress raises the degree of political polarization. Moreover, if technological progress is capital-biased, it raises each voter's ideal level of capital income tax rate and lowers each voter's ideal level of average marginal labor income tax rate, while it does not affect any voter's ideal degree of labor income tax progressivity.

For proof, see Appendix A6.

Because a voter's ideal degree of labor income tax progressivity is set by the voter's earning ability and the earning-ability distribution which are unaffected by an increase in α , capital-biased technological progress does not affect any voter's ideal degree of labor income tax progressivity. On the other hand, because biased technological progress alters relative pre-tax prices of capital and labor to change input-price distortions and relative incentives for supplying capital and labor, voters adjust their own ideal levels of capital and labor income tax rates. In particular, by raising the relative capital productivity, capital-biased technological progress makes relatively less capital necessary to produce one unit of output for public goods and private goods. Thus, when technological progress is capital-biased, it alters the trade off between public-goods benefit and voters' disincentives for supplying capital and labor. It is straightforward from (5) that capital-biased technological progress (an increase in α) lowers the price elasticity of capital and raises the price elasticity of labor. For any given utility gain from public-goods provision, voters prefer minimizing efficiency loss from the

disincentives. Therefore, capital-biased technological progress causes each voter to support higher rate of capital income tax and lower rate of labor income tax rate.

While all voters agree on the directions that the capital and labor income tax rate changes should take for responding to capital-biased technological progress, voters do not agree on the ideal magnitudes of the income tax rate changes. Because voters are different in terms of their ability to earn labor income, a given amount of reduction in marginal labor income tax rate yields higher post-tax rate of return to one unit of labor supply for higher ability voters, whereas a given amount of change in capital income tax rate entails the same change in post-tax rate of return to one unit of capital for all voters. Therefore, capital-biased technological progress causes voters to be more divided on how much to lower the level of average marginal labor income tax rate, while it generates no further disagreement over the ideal rate of capital income tax. In particular, right-wing voters (high-ability voters) prefer to lower the level of average marginal labor income tax rate by *larger margin* than left-wing voters (low-ability voters) do. Putting together, capital-biased technological progress widens the distance between left-wing and right-wing voters' ideal taxation policies, making this economy more polarized politically. Hence, capital-biased technological progress exacerbates political polarization, as opposed to unbiased technological progress.

By the same logic, it is straightforward from *Proposition 4* that labor-biased technological progress (i. e., a decrease in α) alleviates political polarization. When technological progress is labor-biased, it preserves any voter's ideal degree of labor income tax progressivity, as capital-biased or biased technological progress does. However, labor-biased technological progress lowers each voter's ideal level of capital income tax rate and raises each voter's ideal level of average marginal labor income tax rate, with lowering the distance between left-wing and right-wing voters' ideal taxation policies.

In addition, from *Proposition 3 and 4* that state how technological progress affects each voter's ideal taxation policies, including the median voter's, we also can see the effects of technological progress on the politically optimal set of policies. Due to Median Voter Theorem, the set of policies selected democratically by the majority-rule voting coincides with the median voter's ideal set of policies. In light of *Proposition 3*, unbiased technological progress does not affect the politically optimal set of the government policies. In contrast, *Proposition 4* implies that capital-biased technological progress raises the politically optimal rate of tax on capital income and lowers the politically optimal rate of average marginal labor income tax without affecting the politically optimal degree of labor income tax progressivity. As a corollary of *Proposition 4*, labor-biased technological progress lowers the politically optimal rate of tax on capital income and raises the politically optimal rate of average marginal labor income tax without affecting the politically optimal degree of labor income tax progressivity. In light of *Lemma 1 and 2*, changes in the politically optimal set of the government policies resulting from technological progress entail changes in competitive-equilibrium allocation as well.

Notice that this paper discovers the mechanism by which production technology advancements affect political polarization. *Proposition 3 and 4* together show that the effects on

political polarization of technological progress depend on the relative factor productivity. If technological progress is unbiased with preserving the relative factor productivity, it does not affect political polarization. Only if technological progress is capital-biased with raising the relative capital productivity, it exacerbates political polarization by altering voters' relative incentives for supplying labor and capital. By unevenly affecting left-wing and right-wing voters' ideal taxation policies, capital-biased technological progress widens the distance between left-wing and right-wing voters' ideal policies. Different from the existing studies, this paper shows that even *without* any change in preference parameters or in the parameters of σ_ε and σ_π (that represent income inequality), technological progress makes voters more polarized politically if it is capital-biased. Moreover, for tractability, this paper assumes linear capital income tax which results in no disagreement over capital income tax rate but enables us to find out the mechanism by which technological progress affects political polarization. With the logic of the mechanism, allowing nonlinear capital income tax, as a future extension of this study, would still find that technological progress makes a difference in political polarization, although it will make a closed-form solution like ours infeasible to obtain.

6. Effect of Technological Progress on Politico-Economic Distortion

This section analyzes whether technology-driven political polarization affects politico-economic output distortion. Because each voter cares only about their own utility and does not internalize other voters' utility or the social welfare ($SW = \int_{\Theta} U_i dF_{\Theta}$ population-weighted sum of the utility of all voters of this economy) at all, voters' choice of the government policies by the majority-rule voting is different from the government policies under the dictatorship of the social planner who maximizes the social welfare. Thus, the total output under the politically optimal set of policies is different from the total output under the socially optimal set of policies (i. e., set of policies that maximizes the social welfare). The former is politically optimal total output and the latter is socially optimal total output. Although the benevolent social planner does not exist in reality, the socially optimal total output sets benchmark for conceivable first-best total output. In this light, difference between the politically optimal output and the socially optimal output is regarded as politico-economic output distortion. In other words, the fact that the government policies are politically decided by voters (none of whom seeks to maximize the social welfare), instead of by the benevolent social planner, distorts the economy by preventing the politically optimal output from reaching the socially optimal output. Thus, the degree of politico-economic output distortion is indicated by the ratio of these two outputs

$$\delta = \frac{Y^{s*}}{Y^{p*}}, \quad (30)$$

where Y^{s*} is socially optimal total output and Y^{p*} is politically optimal total output. By definition, $\delta > 1$ means existence of politico-economic output distortion, whereas $\delta = 1$ means no such distortion.

Before we examine how this output distortion is affected by technology-driven political polarization, we begin with showing the existence of politico-economic output distortion in the baseline economy before any technological progress occurs.

Lemma 3. The politically optimal total output is strictly smaller than the socially optimal total output. That is, $\delta > 1$.

For proof, see Appendix A7.

Basically, unlike the benevolent social planner, any voter does not seek to increase other voters' post-tax incomes or the social welfare, which is also true for the median voter who plays a decisive role in the majority-rule voting. As a result of this failure in internalizing others' utility (or the social welfare) for making voting decisions, politically decided policies by voting induce smaller total output than the social planner's policies can induce.

To see whether technological progress makes a difference in the degree of politico-economic output distortion, we first introduce an increase in z_T , z_K and z_L for examining the effect of unbiased technological progress on politico-economic output distortion.

Proposition 5. Unbiased technological progress does not affect the degree of politico-economic output distortion.

For proof, see Appendix A8.

As *Proposition 3* shows, none of labor-augmenting technological progress, capital-augmenting technological progress and total factor productivity improvement changes any voter's ideal taxation policies. Thus, unbiased technological progress makes no difference in the politically optimal set of government policies as well as in the degree of political polarization. Moreover, because the set of the government policies that maximizes each voter's utility is unaffected by unbiased technological progress, the set of the government policies that maximizes the social welfare is also unaffected by unbiased technological progress. Thus, unbiased technological progress has no impact on the socially optimal set of policies. As a result of no change in the politically optimal policies and the socially optimal policies, unbiased technological progress raises the levels of Y^{S^*} and Y^{D^*} at the same rate so that it does not change the degree of politico-economic output distortion.

Next, we analyze the effect of biased technological progress (a change in α) on politico-economic output distortion.

Proposition 6. Biased technological progress does not affect the degree of politico-economic output distortion.

For proof, see Appendix A9.

As shown in *Proposition 4* and its application for labor-biased technological progress as a corollary, biased technological progress changes *all* voters' ideal rates of capital and

labor income tax, including the pivotal median voter's, toward the same directions without affecting their ideal degree of labor income tax progressivity. As a result, both capital-biased and labor-biased technological progresses change the politically optimal tax rates to the same direction as they do the socially optimal tax rates without altering politically or socially optimal degree of labor income tax progressivity, although the two sets of optimal taxation policies keep being different from each other, as shown in the proof of *Proposition 6*. Notably, the socially optimal degree of labor income tax progressivity is different from the politically optimal degree, because the benevolent social planner cares about income redistribution for the social welfare gain, in contrast to the median voter as well as all the other voters who do not but selfishly cares his own utility. Fundamentally, biased technological progress does not affect the source of politico-economic output distortion, which is voters' failure to internalize others' utility and the social welfare in their own voting decision, while biased technological progress has the political consequence of changing ideological distance between right-wing and left-wing voters as shown in the previous section. As a result, capital-biased technological progress does not exacerbate the existing politico-economic output distortion, like unbiased technological progress, although capital-biased technological progress exacerbates political polarization, as opposed to unbiased technological progress.

Most of all, *Proposition 5 and 6* together show that technology-driven political polarization does not affect politico-economic output distortion. When political polarization originates from technological progress, it does not entail more distortion on output. To be exact, nonetheless, this paper does not prove that political polarization never affects politico-economic output distortion. In addition, in relation to Alesina and Tabellini (1990) and Azzimonti and Talbert (2014), the model of this paper can be extended to analyze other economic consequences of technology-driven political polarization with respect to government debt and business-cycle fluctuation, which is not of the current focus but delegated to a future study.

7. Quantitative Analysis

This section conducts simulation analyses to quantify the effects of technological progress on political polarization. To this end, the model of this paper is calibrated to the US data. For estimating the technology parameters, we use data over 1970-1979 for the baseline economy and data after 2011 for the economy after technological progress. Utilizing income share data from Federal Reserve Bank, the parameter of α is set as 0.367 for the baseline economy, which is later elevated to 0.405 for simulating capital-biased technological progress. This increase in α is consistent with Elsby *et al.* (2013) and Karabarbounis and Neiman (2014). Since the US Federal Reserve Bank assumes that $z_K = 1$ and $z_L = 1$ for estimating total factor productivity, the data of total factor productivity that it provides is not exactly corresponding to z_T of our model. On the other hand, data of z_K and z_L or their direct estimate is not available. In this light, the values of z_T , z_K and z_L are set as 1 for the baseline economy. Later, for simulating unbiased technological progress, the value of z_T , z_K and z_L is increased respectively. The value of β is calibrated to beget the aggregate marginal product of capital as 0.09, which is the estimate of Caselli and Feyrer (2007) that takes into account of returns

from reproducible capital. The value of η is set as 0.2 following the estimate of Peterman (2016)⁸. The parameter of σ_π is calibrated to yield the Gini index of pre-tax income as 0.466 that is the average Gini index of the US pre-tax income over 1970-2016 (Solt, 2016).

Based on the OECD data, the US government revenue from taxing capital incomes of individuals is 0.4% of GDP on average before 1980. In turn, because $\tau_K \alpha Y$ is the level of capital income tax revenue of the economy, $\alpha = 0.367$ implies that the calibrated capital income tax rate of the baseline economy is equal to 0.01. This entails $\beta = 0.918$ because the Euler equation (16) is $1 = \beta[1 + r(1 - \tau_K)]$ at the steady state. Admittedly, this capital income tax rate (and the paired value of β) may look rather low in the light of US capital income tax rate schedule. However, notice that vast majority of the US citizens do not have to pay capital income tax because the minimum level of taxable capital income is fairly high. That is, actual capital income tax rate for the vast majority is 0% so that the effective capital income tax rate for the entire population ends up with much lower than the lowest marginal rate. According to Tax Foundation, the total amount of Federal capital income tax revenue of 1978 was 9,104 million dollars which is only 0.39% of US GDP of the same year, which is translated into our model as 0.0039. In this light, our calibrated capital income tax rate 0.001 is not too low.

The value of χ is calibrated to beget the politically optimal rate of capital income tax as 0.01, according to (A29) in the proof of *Lemma 3*. In addition, we estimate the degree of labor income tax progressivity of the baseline economy as 0.101, utilizing Panel Study of Income Dynamics (PSID) data and NBER's TAXSIM program of the US.⁹ The value of σ_ϵ is calibrated as 3.363 to get 0.101 as the politically optimal degree of labor income tax progressivity, according to (A31) in the proof of *Lemma 3*.

With the calibrated parameters that are displayed in Table 1, we obtain the degrees of political polarization and politico-economic output distortion of the baseline economy in Table 2. The distance between ideal sets of taxation policies of the top 10% income earner (right-wing voter) and the bottom 10% income earner (left-wing voter) is measured for the degree of political polarization, according to (28). For consistency, we keep measuring the distance between these two voters' ideal set of taxation policies after introducing technological progresses. From the distribution of ideal taxation policies of voters, displayed in Figure 3, 4 and 5, we can alternatively calculate distance between any other left-wing and right-wing voters' ideal set of taxation policies to alternatively measure the degree of political polarization. With other left-wing and right-wing voters, the results do not change in a meaningful way.

In addition, based on (A30) in the proof of *Lemma 3*, the politically optimal rate of average marginal labor income tax of the baseline economy is calculated as 28% reported in the bottom panel of Table 2. Also, according to the proof of *Lemma 3*, we calculate the total outputs under the politically optimal set of policies and the socially optimal set of policies, which are clearly different from each other, to identify the degree of politico-economic output distortion. From this, we find that the government policies chosen democratically by the majority-rule voting lead total output to be smaller by 0.4% points than the benevolent social planner would lead.

Table 1
PARAMETERS OF THE BASELINE ECONOMY

α	$z_T(z_K z_L)$	β	η	σ_π	σ_ε	χ
0.367	1.000	0.918	0.200	0.635	3.363	20.991

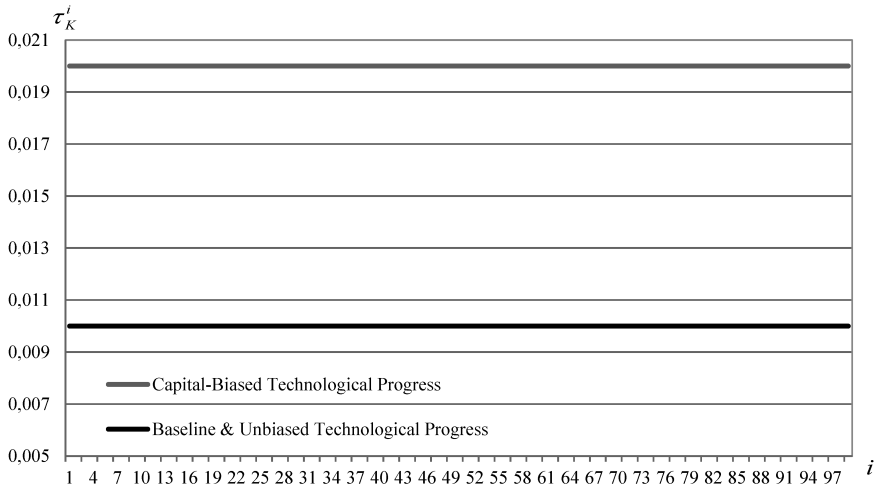
Table 2
POLITICAL POLARIZATION AND DISTORTION OF THE
BASELINE ECONOMY

Political Polarization (ϕ)	0.194
Politico-Economic Output Distortion (δ)	1.004
Capital income tax rate (τ_K^{p*})	0.01
Progressivity of labor income tax (μ_L^{p*})	0.101
Average marginal labor income tax rate (τ_L^{p*})	0.28

Now, unbiased technological progress is introduced by increasing one of the three technology parameters of z_T , z_K and z_L . Specifically, with all the other parameters being fixed, we increase the value of z_T from 1 to 1.273 to represent the increase in the average total factor productivity of the US over 2011-2017 from the average total factor productivity over 1970-1979. Then, for the new steady state reached after this unbiased technological progress, we calculate each voter's ideal taxation policies and display them in Figure 3, 4 and 5, from which we obtain the post-progress degree of political polarization. In Table 3, after the unbiased technological progress of increasing z_T , we report the post-progress degree of political polarization as well as the post-progress degree of politico-economic output distortion from re-calculating the politically optimal set of policies and the socially optimal set of policies with the new value of z_T .

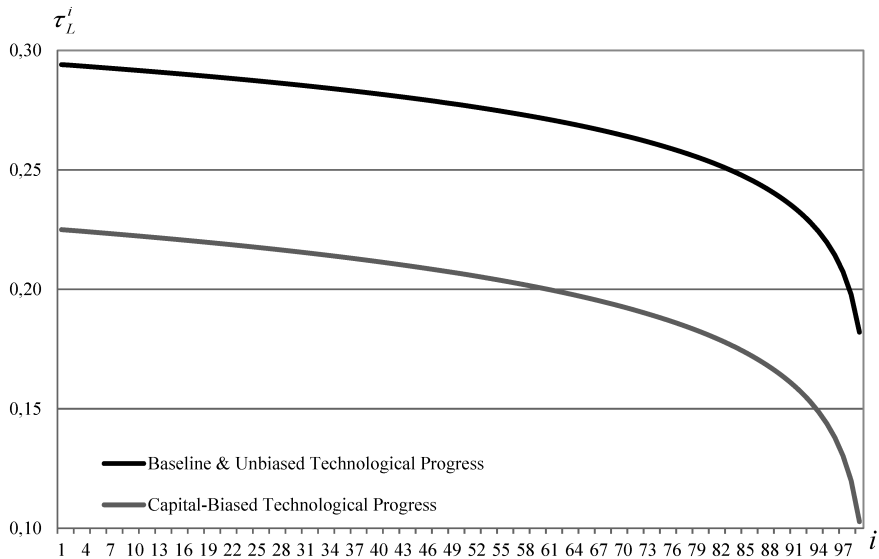
Instead of increasing the value of z_T , we increase the value of z_K to 1.930 from 1 (capital-augmenting technological progress) to generate the same output growth that the increase in z_T yields, with all the other parameters the same as in the baseline economy (Table 1). Likewise, we also increase the value of z_T to 1.464 from 1 (labor-augmenting technological progress) with the baseline parameters to entail the same output growth. These two simulations yield exactly the same results as the simulation of increasing z_T to 1.273 (total factor productivity improvement). Thus, the results of these three simulations of unbiased technological progresses are reported once in Table 3 for efficient use of space. As illustrated in the black lines of Figure 3, 4 and 5, the three unbiased technological progresses do not change any voter's ideal set of taxation policies, as they do not alter the relative factor productivity. Thus, none of capital-augmenting technological progress, labor-augmenting technological progress and total factor productivity improvement affects political polarization or politico-economic output distortion, as shown in the last column of Table 3.

Figure 3
DISTRIBUTION OF VOTERS' IDEAL RATES OF CAPITAL INCOME TAX



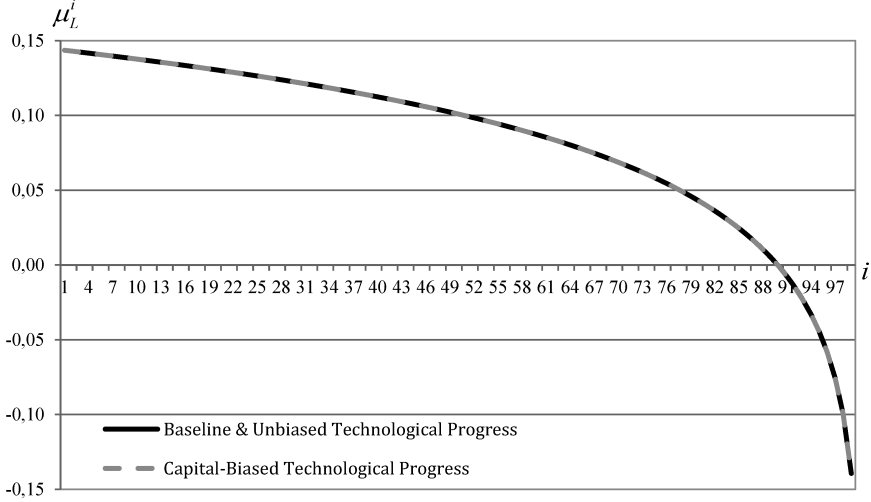
Note: The axis of i represents the percentile of earning ability of voters so that the 50 indicates the median voter. Higher value of i means higher level of earning ability.

Figure 4
DISTRIBUTION OF VOTERS' IDEAL RATES OF AVERAGE MARGINAL LABOR INCOME TAX



Note: The axis of i represents the percentile of earning ability of voters so that the 50 indicates the median voter. Higher value of i means higher level of earning ability.

Figure 5
DISTRIBUTION OF VOTERS' IDEAL DEGREES OF LABOR INCOME TAX PROGRESSIVITY



Note: The axis of i represents the percentile of earning ability of voters so that the 50 indicates the median voter. Higher value of i means higher level of earning ability.

Table 3
POLITICAL POLARIZATION AND DISTORTION DUE TO UNBIASED TECHNOLOGICAL PROGRESS

$z_T: 1 \rightarrow 1.273$ (equivalently, $z_K: 1 \rightarrow 1.930$ or $z_L: 1 \rightarrow 1.464$)	Δ	
Political Polarization (ϕ)	0.194	0.00
Politico-Economic Output Distortion (δ)	1.004	0.00
Capital income tax rate (τ_K^{p*})	0.01	0.00
Progressivity of labor income tax (μ_L^{p*})	0.101	0.00
Average marginal labor income tax rate (τ_L^{p*})	0.28	0.00

Note: The last column reports the difference from the corresponding value of the baseline economy to indicate the effect of unbiased technological progress.

Next, we introduce capital-biased technological progress by increasing the technology parameter α to 0.405 from 0.367 with all the other parameters fixed at their baseline levels (Table 1). This capital-biased technological progress raises the relative capital productivity from 0.580 to 0.681, while increasing total output. We calculate each voter's ideal set of taxation policies for the new steady state reached after this capital-biased technological progress. Unlike unbiased technological progress, the capital-biased technological progress does change each voter's ideal set of taxation policies, as demonstrated by the grey lines of Figure

3, 4 and 5. In particular, the increase in α raises all voters' ideal rate of capital income tax but lowers all voters' ideal rates of average marginal labor income tax by different margins, without affecting their own ideal degree of labor income progressivity. As a result, as reported in Table 4, the capital-biased technological progress raises the politically optimal rate of capital income tax to 2% and lowers the politically optimal rate of average marginal labor income tax to 21% without changing the politically optimal degree of labor income tax progressivity.

Table 4
POLITICAL POLARIZATION AND DISTORTION DUE TO CAPITAL-BIASED TECHNOLOGICAL PROGRESS

$\alpha: 0.367 \rightarrow 0.405$		Δ
Political Polarization (ϕ)	1.199	+0.01
Politico-Economic Output Distortion (δ)	1.004	0.00
Capital income tax rate (τ_K^{p*})	0.02	+0.00
Progressivity of labor income tax (μ_L^{p*})	0.101	0.00
Average marginal labor income tax rate (τ_L^{p*})	0.21	-0.07

Note: The last column reports the difference from the corresponding value of the baseline economy to indicate the effect of capital-biased technological progress.

More importantly, as shown in the last column of Table 4, in contrast to the unbiased technological progress (Table 3), the capital-biased technological progress increases the degree of political polarization. While the capital-biased technological progress (the increase in α to 0.405) positively affects political polarization, it does not affect politico-economic output distortion by raising the politically optimal total output and the socially optimal total output at the same rate, as shown in Table 4.

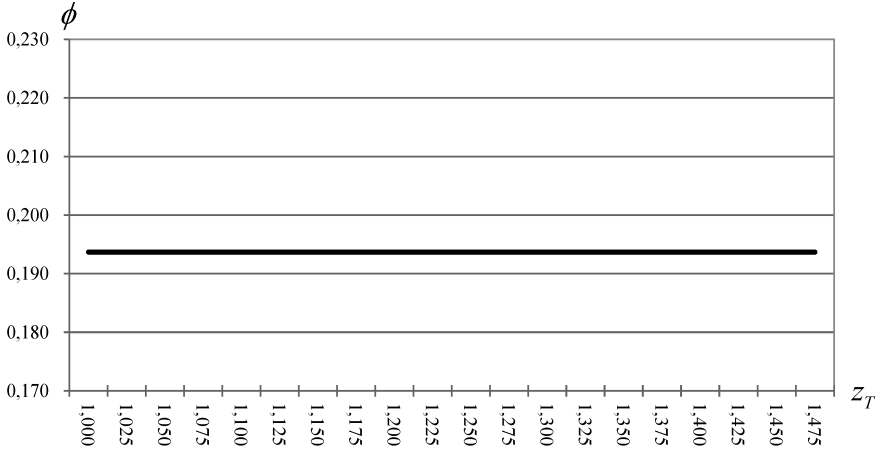
To have more concrete understanding about the magnitude of effect of this capital-biased technological progress on political polarization (1% difference in the left-wing and right-wing voters' ideal rates of average marginal labor income tax)¹⁰, how much 1% change in average marginal labor income tax rate can change total output Y^{p*} is calculated. To this purpose,

from (22) and (26) we obtain that $Y^{p*} = z_T \frac{1}{(1-\alpha)} z_K \frac{\alpha}{(1-\alpha)} z_L \left(\frac{\beta}{1-\beta} \right)^{\frac{\alpha}{(1-\alpha)}} \alpha^{\frac{\alpha}{(1-\alpha)}} (1 - \tau_K^{p*})^{\frac{\alpha}{(1-\alpha)}}$

$\frac{1}{(1-\sigma_\pi)} \left\{ \frac{\chi + 1}{\alpha\chi + 1} (1 - \alpha - \tau_L^{p*}) (1 - \beta) \right\}^{\frac{\eta}{\eta+1}}$. When we deviate from the politically optimal

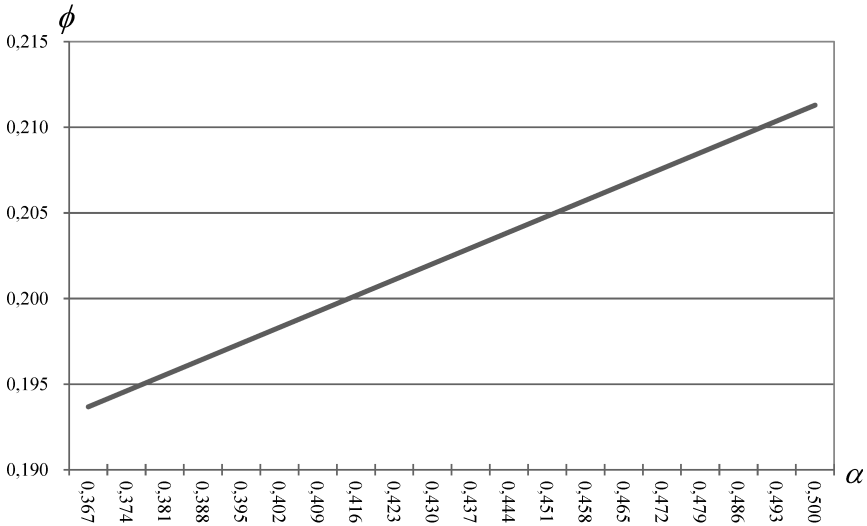
rate of average marginal labor income tax (Table 4) by 1%, we find that the total output changes about 1%. This is not negligible, as it is similar to the total size of corporate income tax revenue of US (1.1% of GDP, or 217.8 billion dollars in 2018), according to the OECD data. Although this paper does not argue that technology or disagreement over taxation policies is the only source of political polarization, the quantitative result of this section suggests that the role of technology progress in the recently increasing political polarization can be substantial.

Figure 6
EFFECT OF UNBIASED TECHNOLOGICAL PROGRESS ON POLITICAL POLARIZATION



Note: Instead of increasing z_T from 1 to 1.475, increasing z_K from 1 to 2.884 or increasing z_L from 1 to 1.848 yields exactly the same result of the effect on political polarization as above, while entailing the same growth of total output.

Figure 7
EFFECT OF CAPITAL-BIASED TECHNOLOGICAL PROGRESS ON POLITICAL POLARIZATION



In addition, we further examine the effects of technological progress on political polarization by increasing the technology parameters to other feasible values. First, Figure 6

illustrates that the degree of political polarization remains unaltered when the value of z_T continuously increases from 1 to 1.475, showing no effect of unbiased technological progress on political polarization. Second, Figure 7 reports the quantitative effects of capital-biased technological progress on political polarization. As capital-biased technological progress is intensified (i. e., as the value of α increases from 0.367 to 0.500), it widens the distance between the left-wing and right-wing voters' ideal taxation policies by 0.02 more. Moreover, Table 3 and 4 as well as Figure 6 and 7 together suggest that although the US economy simultaneously experienced both unbiased technological progress and capital-biased technological progress after 1976 (Figure 1), the post-1976 increase in the degree of political polarization (Figure 2) is attributed to capital-biased technological progress, not to unbiased technological progress. Notably, as Figure 1 demonstrates, while the relative capital productivity (capital income share) did not clearly increase before 1976, total factor productivity clearly increased. At the same time, Figure 2 shows that the degree of political polarization did not increase before 1976.

8. Concluding Remarks

This paper analyzes the effects of technological progress on political polarization in a steady-state economy where individual voters are differentiated by their earning ability. To this end, we obtain each voter's ideal policy of labor and capital income taxation. While a set of the government tax policies is selected democratically by the majority-rule voting, individual voter's ideal set of taxation policies is different from each other's. Based on the obtained voters' ideal policies, we identify the degree of political polarization with the distance between left-wing and right-wing voters' ideal set of policies, where right-wing voters have higher earnings than the median voter and left-wing voters have lower earnings than the median voter. Depending on whether the relative factor productivity changes or not, technological progress is classified as either biased or unbiased technological progress. First, because biased technological progress unequally affects left-wing and right-wing voters' ideal taxation policies by raising the relative capital or labor productivity, technological progress raises the degree of political polarization if it is capital-biased and lowers the degree if it is labor-biased. Nonetheless, such a technology-driven political polarization does not affect politico-economic output distortion. Second, this paper also finds that unbiased technological progress (labor-augmenting technological progress, capital-augmenting technological progress and total factor productivity improvement) does not affect political polarization or politico-economic output distortion, because it does not alter relative incentives to supply labor and capital to make all voters' ideal taxation policies remain unchanged with preserving the relative factor productivity. These two findings together suggest that the effect of technological progress on political polarization depends on the *relative* factor productivity.

Appendix

A1. Proof of Lemma 1

First of all, using the Guess and Verify method, we identify the value function of (4) at a stationary competitive general equilibrium. For any given $\theta_i \in \Theta$, guess that the value function is $v_i(k) = A \log(k) + B$ with unknown A and B . Because the budget constraint (2) is met, using (13), $c_i = (1 + s_i)(rk_i(1 - \tau_K) + k_i - k'_i)$ for any given $\theta_i \in \Theta$. Then, the FOC with respect to capital-investment decision is

$$(1 - \beta) \frac{-1}{(rk_i(1 - \tau_K) + k_i - k'_i)} + \beta A \frac{1}{k'_i} = 0, \quad (\text{A1})$$

which implies that

$$k'_i = \frac{\beta A(r(1 - \tau_K) + 1)}{(1 - \beta + \beta A)} k_i. \quad (\text{A2})$$

Utilizing (4), (13), and (A2), the value function v_i is restated as

$$\begin{aligned} A \log(k_i) + B &= (1 - \beta) \log(1 + s_i) + (1 - \beta) \log\left(\frac{(1 - \beta)(r(1 - \tau_K) + 1)k_i}{(1 - \beta + \beta A)}\right) - \\ &(1 - \beta) \frac{l_i^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} + (1 - \beta)\chi \log(G) + \beta A \log\left(\frac{\beta A(r(1 - \tau_K) + 1)k_i}{(1 - \beta + \beta A)}\right) + \beta B. \end{aligned} \quad (\text{A3})$$

Comparing the coefficient of $\log(k_i)$ in the left- and right-hand sides of (A3) yields that $A = 1$, for any given $\theta_i \in \Theta$. Likewise, for any given $\theta_i \in \Theta$, we obtain that

$$\begin{aligned} B &= \log(1 + s_i) - \frac{l_i^{1+\frac{1}{\eta}}}{\left(1 + \frac{1}{\eta}\right)} + \chi \log(G) + \log(r(1 - \tau_K)) + \\ &\frac{1}{1 - \beta} [\log(r(1 - \tau_K) + 1) + \beta \log(\beta)]. \end{aligned} \quad (\text{A4})$$

Because $k_i = k'_i > 0$ takes a finite value at a steady-state competitive general equilibrium, the value function with these finite coefficients of A and B also takes a finite value at a steady-state competitive general equilibrium. Now, with the identified value function v_i , for any given $\theta_i \in \Theta$, the optimal labor-supply condition of (14) is

$$(1 - \beta) \frac{(1 - \mu_L)}{l_i} - (1 - \beta) l_i^{\frac{1}{\eta}} - \beta l_i^{\frac{1}{\eta}} = 0, \quad (\text{A5})$$

which implies that the optimal labor supply of individual voter i is equal to (17) for any given $\theta_i \in \Theta$. At the same time, the optimality condition of (15) is

$$(1 - \beta) \frac{1}{\left(1 + \frac{1}{s_i}\right)} (-1) \left(\frac{1}{s_i}\right)^2 + \beta \frac{1}{(1 + s_i)} = 0, \quad (\text{A6})$$

which entails that the optimal ratio of post-tax labor income to post-tax capital income of individual voter i is equal to (19) for any given $\theta_i \in \Theta$. Moreover, as $c_i = \left(1 + \frac{1}{s_i}\right) \rho_L(w\theta_i l_i)^{1-\mu_L}$ at a stationary competitive general equilibrium, (A5) and (A6) imply that the optimal private consumption of individual voter i is equal to (18) for any given $\theta_i \in \Theta$.

A2. Proof of Lemma 2

First, due to (12) and (17) of *Lemma 1*, the aggregate labor supply at a stationary competitive general equilibrium is

$$L = \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}} \int_{\Theta} \theta dF_{\theta} = \frac{1}{(1 - \sigma_{\pi})} \{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}}, \quad (\text{A7})$$

which shows that the total labor is (20). Second, with (5), (7), (8) and (17), the government's budget constraint of (6) at a stationary competitive general equilibrium is stated as

$$\int_{\Theta} \{w\theta\{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}} - \rho_L(w\theta\{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}})^{1-\mu_L} + \tau_K r k_i(\theta) dF_{\theta}\} = gY. \quad (\text{A8})$$

Because $(1 - \alpha)Y = wL$ and $\alpha Y = rK$, with the conditions (11) and (12) being met at a stationary competitive general equilibrium, (A8) is restated as

$$(1 - \alpha)Y + \tau_K \alpha Y - \int_{\Theta} \rho_L(w\theta\{(1 - \mu_L)(1 - \beta)\}^{\frac{\eta}{\eta+1}})^{1-\mu_L} dF_{\theta} = gY. \quad (\text{A9})$$

Furthermore, at a stationary competitive general equilibrium, $k_i = k'_i$ for any given $\theta_i \in \Theta$; hence, $s_i = \frac{\rho_L(w\theta_i l_i)^{1-\mu_L}}{r k_i (1 - \tau_K)}$. Thus, due to (17) and (19) of *Lemma 1* as well as (12), (A9) is simplified into

$$(1 - \alpha)Y + \tau_K \alpha Y - \left(\frac{1 - \beta}{\beta}\right) \alpha Y + \left(\frac{1 - \beta}{\beta}\right) \tau_K \alpha Y = gY. \quad (\text{A10})$$

As $Y > 0$, we can divide both sides of (A10) by Y without affecting the equality, to obtain that $(1 - \alpha) + \tau_K \alpha - \left(\frac{1 - \beta}{\beta}\right) \alpha + \left(\frac{1 - \beta}{\beta}\right) \tau_K \alpha = g$, which entails (23). Third, at a station-

ary competitive general equilibrium, $k_i(\theta_i) = \frac{\rho_L(w\theta_i l_i)^{1-\mu_L}}{s_i r(1-\tau_K)}$ for any given $\theta_i \in \Theta$, which is aggregated over the entire population to get

$$K = \frac{\rho_L(w)^{1-\mu_L} \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}(1-\mu_L)}}{r(1-\tau_K)} \left(\frac{\beta}{1-\beta} \right) \exp \left[\frac{\sigma_\varepsilon(1-\mu_L)(-\mu_L)}{2} \right] \frac{1}{1-\sigma_\pi(1-\mu_L)} \quad (\text{A11})$$

due to (11). The inter-temporal optimality condition of (16) at a stationary competitive general equilibrium, where $k_i = k'_i = k''_i$ for any given $\theta_i \in \Theta$, implies that

$$r = \left(\frac{1-\beta}{\beta} \right) \frac{1}{(1-\tau_K)}. \quad (\text{A12})$$

Moreover, at a stationary competitive general equilibrium, $w = z_T z_L (1-\alpha) \left(\frac{z_K K}{z_L L} \right)^\alpha$. Plugging this market-clearing wage rate, (A12) and (A7) into (A11), and then solving for K entails

$$K = \rho_L^{\frac{1}{1-\alpha(1-\mu_L)}} z_T^{\frac{(1-\mu_L)}{1-\alpha(1-\mu_L)}} (z_K)^{\frac{\alpha(1-\mu_L)}{1-\alpha(1-\mu_L)}} (z_L)^{\frac{(1-\mu_L)(1-\alpha)}{1-\alpha(1-\mu_L)}} \left(\frac{\beta}{1-\beta} \right)^{\frac{2}{1-\alpha(1-\mu_L)}} (1-\alpha)^{\frac{1-\mu_L}{1-\alpha(1-\mu_L)}} \left(\frac{1}{1-\sigma_\pi} \right)^{\frac{-\alpha(1-\mu_L)}{1-\alpha(1-\mu_L)}} \{(1-\mu_L)(1-\beta)\}^{\frac{\eta(1-\mu_L)(1-\alpha)}{(\eta+1)\{1-\alpha(1-\mu_L)\}}} \left(\frac{1}{1-\sigma_\pi(1-\mu_L)} \right)^{\frac{1}{1-\alpha(1-\mu_L)}} \exp \left[\frac{\sigma_\varepsilon(1-\mu_L)(-\mu_L)}{2} \right]^{\frac{1}{1-\alpha(1-\mu_L)}}. \quad (\text{A13})$$

Now, to solve for ρ_L , as $\tau_K r k_i = r k_i - \frac{1}{s_i} \rho_L(w\theta_i l_i)^{1-\mu_L}$ at a stationary competitive general equilibrium, with (17), (A8) is simplified to

$$\left(\frac{1}{1-\beta} \right) \int_{\Theta} \rho_L \left(w\theta \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}} \right)^{1-\mu_L} dF_\theta = (1-g)Y. \quad (\text{A14})$$

Using (5) and (23), (A14) becomes

$$\rho_L \left(\frac{1}{1-\beta} \right) \{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1}(1-\mu_L)} \left(z_T z_L (1-\alpha) \left(\frac{z_K K}{z_L L} \right)^\alpha \right)^{1-\mu_L} \exp \left[\frac{\sigma_\varepsilon(1-\mu_L)(-\mu_L)}{2} \right] \frac{1}{1-\sigma_\pi(1-\mu_L)} = \frac{\alpha}{\beta} (1-\tau_K) z_T (z_K K)^\alpha (z_L L)^{1-\alpha}. \quad (\text{A15})$$

Utilizing (A7), (A13) and (A15), we solve for ρ_L to obtain

$$\rho_L = z_T \frac{\mu_L}{(1-\alpha)} z_K \frac{\alpha \mu_L}{(1-\alpha)} z_T^{\mu_L} \left(\frac{\beta}{1-\beta} \right)^{\frac{\alpha \mu_L - 1 + \alpha}{(1-\alpha)}} \alpha^{\frac{1-\alpha(1-\mu_L)}{1-\alpha}} \left(\frac{1}{1-\alpha} \right)^{(1-\mu_L)} (1-\tau_K)^{\frac{1-\alpha(1-\mu_L)}{1-\alpha}} \quad (\text{A16})$$

$$\{(1-\mu_L)(1-\beta)\}^{\frac{\eta}{\eta+1} \mu_L} \exp \left[\frac{\sigma_\varepsilon(1-\mu_L)(\mu_L)}{2} \right] \frac{\{1-\sigma_\pi(1-\mu_L)\}}{(1-\sigma_\pi)}.$$

Plugging (A16) into (A13) entails (21). Fourth, applying (A7) and (21) to the production function of (5) shows that the aggregate output at a stationary competitive general equilibrium is equal to (22).

A3. Proof of Proposition 1

First, the utility function of (24) for each voter is concave in τ_K for $\forall \tau_K \in [0,1]$. Thus, for any given $\theta_i \in \Theta$, the FOC with respect to τ_K is sufficient to define the ideal rate of capital income tax τ_K^i that maximizes the utility of individual voter i (voter whose earning ability is θ_i).

$$\frac{(\alpha\chi + 1)}{(1-\alpha)} \frac{-1}{(1-\tau_K^i)} + \frac{\chi}{\frac{\beta}{\alpha} - (1-\tau_K^i)} = 0. \quad (\text{A17})$$

Solving (A17) for τ_K^i , we get

$$\tau_K^i = 1 - \frac{\beta}{\alpha} \left(\frac{\alpha\chi + 1}{\chi + 1} \right). \quad (\text{A18})$$

Because the utility function of (24) is concave in τ_K , when $\alpha(\chi + 1) \leq \beta(\alpha\chi + 1)$, the highest feasible level of the utility is attained by $\tau_K^i = 0$. On the other hand, when $\alpha(\chi + 1) > \beta(\alpha\chi + 1)$, the right-hand side of (A18) itself is individual voter i 's ideal capital income tax rate (τ_K^i). When the right-hand side of (A18) is greater than zero, it never exceeds one because $\alpha > 0$, $\chi > 0$ and $\beta > 0$. Putting these two cases of $\alpha(\chi + 1) > \beta(\alpha\chi + 1)$ and $\alpha(\chi + 1) \leq \beta(\alpha\chi + 1)$ together entails (25).

Second, as the utility function of (24) for each voter is also concave in μ_L for all the feasible values in the range of (9), μ_L^i is defined fully by the following FOC. Thus, for any given $\theta_i \in \Theta$,

$$-\theta_i - \left(\frac{\eta}{\eta+1} \right) \frac{(1+\chi)}{(1-\mu_L^i)} + \left(\frac{\eta}{\eta+1} \right) (1-\beta) + \frac{\sigma_\pi}{1-\sigma_\pi(1-\mu_L^i)} + \frac{\sigma_\varepsilon(1-2\mu_L^i)}{2} = 0 \quad (\text{A19})$$

which is equivalently stated as (27).

Third, (A16) of *Lemma 2* shows that at a stationary competitive general equilibrium, where the government's budget is balanced, μ_L and τ_K automatically define ρ_L . Hence, according to (A16), the value of ρ_L^i is automatically determined by τ_K^i and μ_L^i from (25) and (A19).

Fourth, recall that the average marginal labor income tax rate τ_L is defined by (10). Based on *Lemma 1 and 2*, plugging (22) of Y and $y_i(\theta) = w\theta_i l_i(\theta_i)$ where $w = z_T z_L (1 - \alpha) \left(\frac{z_K K}{z_L L}\right)^\alpha$ as well as (A16) into (10) yields the average marginal labor income tax rate τ_L at a stationary competitive general equilibrium as follows.

$$\tau_L = (1 - \alpha) - \frac{\alpha}{\beta} (1 - \mu_L)(1 - \tau_K) \quad (\text{A20})$$

which shows how the average marginal labor income tax rate τ_L is automatically determined once μ_L and τ_K are decided at a stationary competitive general equilibrium. As (A20) means $(1 - \tau_K) = \frac{\beta}{\alpha} \frac{1}{(1 - \mu_L)} (1 - \alpha - \tau_L)$, plugging this into the utility function of (24) and then obtaining the FOC of τ_L^i for any given $\theta_i \in \Theta$ yields

$$\frac{(\alpha\chi + 1)}{(1 - \alpha)} \frac{-1}{(1 - \alpha - \tau_L^i)} + \frac{\chi}{(1 - \mu_L^i) - (1 - \alpha - \tau_L^i)} = 0. \quad (\text{A21})$$

Solving (A21) for τ_L^i entails (26).

A4. Proof of Proposition 2

[Step 1]. To show that whenever $\theta_i > \theta_j$, $\mu_L^i < \mu_L^j$, we apply the Implicit Function Theorem to (27) of *Proposition 1* to get

$$\frac{d\mu_L^i}{d\theta_i} = - \frac{-1}{-\frac{\eta}{\eta + 1} \frac{(\chi + 1)}{(1 - \mu_L^i)^2} - \frac{\sigma_\pi^2}{(1 - \sigma_\pi(1 - \mu_L^i))^2} - \sigma_\varepsilon} \quad (\text{A22})$$

which is strictly negative because $\eta > 0$, $\chi > 0$ and $\sigma_\varepsilon > 0$. $\frac{d\mu_L^i}{d\theta_i} < 0$ means that whenever $\theta_i > \theta_j$, $\mu_L^i < \mu_L^j$.

[Step 2.] To show that whenever $\theta_i > \theta_j$, $\tau_L^i < \tau_L^j$, consider any two voters i and j such that $\theta_i > \theta_j$. Then, based on (26) of *Proposition 1*,

$$\tau_L^i - \tau_L^j = \left(\frac{\alpha\chi + 1}{\chi + 1}\right) (\mu_L^i - \mu_L^j) \quad (\text{A23})$$

which is strictly negative due to the above step 1, $\chi > 0$ and $\alpha > 0$.

A5. Proof of Proposition 3

As described in Section 2, unbiased technological progress is represented by an increase in one of the three technology parameters z_T , z_K and z_L . That is, unbiased technological pro-

gress is realized as one of the following three cases: (i) total factor productivity improvement (an increase in z_T), (ii) capital-augmenting technological progress (an increase in z_K) and (iii) labor-augmenting technological progress (an increase in z_L).

First, consider an increase in z_T with all the other parameters being fixed, which represents technological progress of improving total factor productivity. For any given $\theta_i \in \Theta$, the effect of improving total factor productivity on voter i 's ideal rate of capital income tax is identified by $\frac{d\tau_K^i}{dz_T}$. According to (25) of *Proposition 1*, $\frac{d\tau_K^i}{dz_T} = 0$ showing that increasing total factor productivity has no effect on voter i 's ideal rate of capital income tax. By the same token, the effect of improving total factor productivity on voter i 's ideal average marginal labor income tax rate is identified with $\frac{d\tau_L^i}{dz_T}$ which is equal to zero, according to (26) of *Proposition 1*. Thus, unbiased technological progress of increasing total factor productivity does not affect voter i 's ideal level of average marginal labor income tax rate. Likewise, an increase in z_T has no effect on voter i 's ideal degree of labor income tax progressivity, because $\frac{d\mu_L^i}{dz_T} = 0$ due to (27) of *Proposition 1*. This holds for any arbitrarily given $\theta_i \in \Theta$; hence, total factor productivity improvement (an increase in z_T) does not change any voter's ideal levels of capital income tax rate and average marginal labor income tax rate or any voter's ideal degree of labor income tax progressivity. As a result, total factor productivity improvement (an increase in z_T) does not change the ideal taxation policies of left-wing and right-wing voters so that it does not affect the degree of political polarization.

Second, now let us introduce an increase in z_K with all the other parameters being fixed, to represent capital-augmenting technological progress. By the same logic, because $\frac{d\tau_K^i}{dz_K} = 0$, $\frac{d\tau_L^i}{dz_K} = 0$ and $\frac{d\mu_L^i}{dz_T} = 0$ for any arbitrarily given $\theta_i \in \Theta$ according to *Proposition 1*, capital-augmenting technological progress does not change any voter's ideal levels of capital income tax rate and average marginal labor income tax rate or any voter's ideal degree of labor income tax progressivity. Therefore, capital-augmenting technological progress (an increase in z_K) does not change the ideal taxation policies of left-wing and right-wing voters so that it does not affect the degree of political polarization.

Third, consider an increase in z_L with all the other parameters fixed, which represents labor-augmenting technological progress. Labor-augmenting technological progress also makes no difference in any voter's ideal levels of capital income tax rate and average marginal labor income tax rate, because $\frac{d\tau_K^i}{dz_L} = 0$ and $\frac{d\tau_L^i}{dz_L} = 0$ for any given $\theta_i \in \Theta$ due to *Proposition 1*. Likewise, labor-augmenting technological progress does not affect any voter's ideal

degree of labor income tax progressivity, as $\frac{d\mu_L^i}{dz_L} = 0$ for any given $\theta_i \in \Theta$ according to *Proposition 1*. As a consequence, labor-augmenting technological progress does not affect the degree of political polarization, either.

Taking the above three cases together shows that unbiased technological progress does not affect the degree of political polarization.

A6. Proof of Proposition 4

[Step 1.] As noted in the Section 2, capital-biased technological progress is represented by an increase in α with all the other parameters being fixed. For any given $\theta_i \in \Theta$, the effect of capital-biased technological progress on voter i 's ideal level of capital income tax rate is identified with $\frac{d\tau_K^i}{d\alpha}$. If $\alpha(\chi + 1) > \beta(\alpha\chi + 1)$ before capital-biased technological progress, according to (25) of *Proposition 1*, capital-biased technological progress strictly raises voter i 's ideal level of capital income tax rate for any given $\theta_i \in \Theta$, because

$$\frac{d\tau_K^i}{d\alpha} = \frac{\beta}{\alpha^2(\chi + 1)} > 0. \quad (\text{A24})$$

Moreover, as α increases, the condition for τ_K^i to be strictly positive is more likely to be satisfied, because

$$\frac{d\{\alpha(\chi + 1) - \beta(\alpha\chi + 1)\}}{d\alpha} = (1 - \beta)\chi + 1 > 0. \quad (\text{A25})$$

If $\alpha(\chi + 1) \leq \beta(\alpha\chi + 1)$ before capital-biased technological progress, (A24) and (A25) imply that capital-biased technological progress either strictly raises voter i 's ideal level of capital income tax rate to be strictly positive or maintains it as zero. Because ideal level of capital income tax rate is equal for all voters, the majority-rule voting outcome on the capital income tax rate will be equal to voter i 's ideal level of capital income tax rate. Therefore, capital-biased technological progress raises each voter's ideal level of capital income tax rate.

[Step 2.] For any given $\theta_i \in \Theta$, applying the Implicit Function Theorem to (27) of *Proposition 1*, the effect of capital-biased technological progress on voter i 's ideal degree of labor income tax progressivity is identified with

$$\frac{d\mu_L^i}{d\alpha} = 0. \quad (\text{A26})$$

Because (A26) holds for any arbitrarily given $\theta_i \in \Theta$, capital-biased technological progress does not affect any voter's ideal degree of labor income tax progressivity.

[Step 3.] For any given $\theta_i \in \Theta$, the effect of capital-biased technological progress (an increase in α) on voter i 's ideal level of average marginal labor income tax rate is indicated by $\frac{d\tau_L^i}{d\alpha}$. Based on (26) of *Proposition 1*,

$$\frac{d\tau_L^i}{d\alpha} = (-1) - (1 - \mu_L^i) \frac{\chi}{(\chi + 1)} < 0 \quad (\text{A27})$$

because (A26), (9) and $\chi > 0$. As (A27) holds for any arbitrarily given $\theta_i \in \Theta$, capital-biased technological progress lowers each voter's ideal rate of average marginal labor income tax.

[Step 4.] Having found the effect of capital-biased technological progress on each voter's ideal taxation policies, this step examines the effect of capital-biased technological progress on distance between any two different voters' ideal taxation policies so that we can identify the effect of capital-biased technological progress on political polarization. To this end, first, due to all voters' agreement over the ideal level of capital income tax rate, (25) of *Proposition 1*, $|\tau_K^i - \tau_K^j| = 0$ for any given i and j , before and after an increase in α . Second, because capital-biased technological progress does not affect any voter's ideal degree of labor income tax progressivity (as shown in the step 2 above), it does not affect $|\mu_L^i - \mu_L^j|$. Third, according to (A23) in the proof of *Proposition 2*,

$$|\tau_L^i - \tau_L^j| = \left(\frac{\alpha\chi + 1}{\chi + 1} \right) |(\mu_L^i - \mu_L^j)| \quad (\text{A28})$$

which strictly increases with α . That is, capital-biased technological progress (an increase in α) strictly increases the value of $|\tau_L^i - \tau_L^j|$ for any given i and j . Combining these changes in the distance between any two different voters' ideal taxation policies caused by an increase in α shows that capital-biased technological progress widens distance between any two different voters' ideal taxation policies.

[Step 5.] The above step 4 holds for any two different voters; hence, it also holds for left-wing and right-wing voters. Therefore, capital-biased technological progress strictly increases $|\tau_L^{left} - \tau_L^{right}|$ while preserving the value of $|\tau_K^{left} - \tau_K^{right}| + |\mu_L^{left} - \mu_L^{right}|$. Thus, capital-biased technological progress raises the degree of political polarization.

Notice that each step of this proof holds regardless of whether the condition of (29) is met or not.

A7. Proof of Lemma 3

[Step 1.] We begin with identifying the politically optimal set of policies. As mentioned in Section 3, because the indirect utility function of each voter, (24), satisfies intermediate preference, we can apply the Median Voter Theorem to identify the politically optimal set of

policies with the ideal set of policies of the voter who is born with median-value ability (the median voter). Hence, according to *Proposition 1*, the set of the politically optimal set of policies $(\tau_K^{p*}, \tau_L^{p*}$ and $\mu_L^{p*})$ is defined as follows.

$$\tau_K^{p*} = \begin{cases} 1 - \frac{\beta}{\alpha} \left(\frac{\alpha\chi + 1}{\chi + 1} \right) & \text{if } \alpha(\chi + 1) > \beta(\alpha\chi + 1) \\ 0 & \text{if } \alpha(\chi + 1) \leq \beta(\alpha\chi + 1) \end{cases}, \quad (\text{A29})$$

$$\tau_L^{p*} = (1 - \alpha) - (1 - \mu_L^{p*}) \left(\frac{\alpha\chi + 1}{\chi + 1} \right), \quad (\text{A30})$$

$$\frac{\sigma_\pi}{1 - \sigma_\pi(1 - \mu_L^{p*})} + \sigma_\varepsilon(1 - \mu_L^{p*}) - \sigma_\pi \log(2) = \frac{\eta}{\eta + 1} \frac{(1 + \chi)}{(1 - \mu_L^{p*})} - \frac{\eta}{\eta + 1} (1 - \beta). \quad (\text{A31})$$

Based on (A29), (A30) and (A31), (22) of *Lemma 2* identifies the politically optimal total output Y^{p*} as follows:

$$Y^{p*} = z_T^{\frac{1}{(1-\alpha)}} z_K^{\frac{\alpha}{(1-\alpha)}} z_L \left(\frac{\beta}{1-\beta} \right)^{\frac{\alpha}{(1-\alpha)}} \alpha^{\frac{\alpha}{(1-\alpha)}} (1 - \tau_K^{p*})^{\frac{\alpha}{(1-\alpha)}} \frac{1}{(1 - \sigma_\pi)} \{(1 - \mu_L^{p*})(1 - \beta)\}^{\frac{\eta}{\eta+1}}. \quad (\text{A32})$$

[Step 2.] To identify the socially optimal set of policies that maximizes the social welfare $SW = \int_{\Theta} U_i dF_{\Theta}$, we aggregate the indirect utility function of (24) over the entire population to yield the social welfare function as

$$\begin{aligned} SW &= (1 + \chi) \left\{ \frac{1}{(1 - \alpha)} \log(z_T) + \log(z_L) + \frac{\alpha}{(1 - \alpha)} \log(z_K) \right\} + \\ &(1 - \mu_L) \left(\sigma_\pi - \frac{\sigma_\varepsilon}{2} \right) + (\chi + 1) \frac{\eta}{\eta + 1} \log\{(1 - \mu_L)(1 - \beta)\} - \frac{\eta\{(1 - \mu_L)(1 - \beta)\}}{\eta + 1} + \\ &\left(\frac{\alpha\chi + 1}{1 - \alpha} \right) \log(1 - \tau_K) + \chi \log \left(1 - \frac{\alpha}{\beta} (1 - \tau_K) \right) + \log \left(\frac{1 - \sigma_\pi(1 - \mu_L)}{1 - \sigma_\pi} \right) + \\ &\frac{\sigma_\varepsilon \mu_L (1 - \mu_L)}{2} + \frac{\alpha(\chi + 1)}{(1 - \alpha)} \log \left(\frac{\beta}{1 - \beta} \right) - \log(\beta) + \frac{\alpha\chi + 1}{(1 - \alpha)} \log(\alpha) - \chi \log(1 - \sigma_\pi). \end{aligned} \quad (\text{A33})$$

Maximizing the social welfare function of (A33) obtains the the set of socially optimal taxation policies $(\tau_K^s, \tau_L^s$ and $\mu_L^s)$ as follows.

$$\tau_K^{s^*} = \begin{cases} 1 - \frac{\beta}{\alpha} \left(\frac{\alpha\chi + 1}{\chi + 1} \right) & \text{if } \alpha(\chi + 1) > \beta(\alpha\chi + 1) \\ 0 & \text{if } \alpha(\chi + 1) \leq \beta(\alpha\chi + 1) \end{cases}, \quad (\text{A34})$$

$$\tau_L^{s^*} = (1 - \alpha) - (1 - \mu_L^{s^*}) \left(\frac{\alpha\chi + 1}{\chi + 1} \right), \quad (\text{A35})$$

$$\frac{\sigma_\pi}{1 - \sigma_\pi(1 - \mu_L^{s^*})} + \sigma_\varepsilon(1 - \mu_L^{s^*}) - \sigma_\pi = \frac{\eta}{\eta + 1} \frac{(1 + \chi)}{(1 - \mu_L^{s^*})} - \frac{\eta}{\eta + 1} (1 - \beta). \quad (\text{A36})$$

Notice that allocation of labor supply and savings are chosen by individuals themselves, not by a social planner, and implemented as a stationary competitive general equilibrium of this economy. Thus, according to (22) of *Lemma 2*, the socially optimal total output Y^{s^*} is

$$Y^{s^*} = z_T \frac{1}{(1-\alpha)} z_K \frac{\alpha}{(1-\alpha)} z_L \left(\frac{\beta}{1-\beta} \right)^{\frac{\alpha}{(1-\alpha)}} \alpha^{\frac{\alpha}{(1-\alpha)}} (1 - \tau_K^{s^*})^{\frac{\alpha}{(1-\alpha)}} \frac{1}{(1 - \sigma_\pi)} \{(1 - \mu_L^{s^*})(1 - \beta)\}^{\frac{\eta}{\eta+1}}. \quad (\text{A37})$$

In light of the proof of *Proposition 2*, (A31) and (A36), because $\sigma_\pi > \sigma_\pi \log(2)$,

$$\mu_L^{p^*} > \mu_L^{s^*} \quad \text{and} \quad \tau_L^{p^*} > \tau_L^{s^*}. \quad (\text{A38})$$

Thus, there exists a gap between the socially optimal policies and the politically optimal policies.

[Step 3.] Based on (A32), (A37) and $\tau_K^{s^*} = \tau_K^{p^*}$ from (A29) and (A34), we obtain the politico-economic output distortion as

$$\delta = \frac{Y^{s^*}}{Y^{p^*}} = \frac{\{(1 - \mu_L^{s^*})(1 - \beta)\}^{\frac{\eta}{\eta+1}}}{\{(1 - \mu_L^{p^*})(1 - \beta)\}^{\frac{\eta}{\eta+1}}}. \quad (\text{A39})$$

Notably, (A38) implies that $\delta > 1$.

A8. Proof of Proposition 5

[Step 1.] As shown in the proof of *Proposition 3* (Appendix A5), with all the other parameters being fixed, an increase in any one of the three technology parameters z_T , z_K and z_L does not change any voters' ideal taxation policies, including the median voters'. Due to

$$(A29), (A30) \text{ and } (A31) \text{ in the proof of } \textit{Lemma 3}, \frac{d\tau_K^{p^*}}{dz_T} = 0, \frac{d\tau_L^{p^*}}{dz_T} = 0, \frac{d\mu_L^{p^*}}{dz_T} = 0; \frac{d\tau_K^{p^*}}{dz_K} = 0,$$

$\frac{d\tau_L^{p^*}}{dz_K} = 0, \frac{d\mu_L^{p^*}}{dz_K} = 0; \frac{d\tau_K^{p^*}}{dz_L} = 0, \frac{d\tau_L^{p^*}}{dz_L} = 0$ and $\frac{d\mu_L^{p^*}}{dz_L} = 0$. Therefore, unbiased technological progress (an increase in z_T, z_L or z_K) does not affect the politically optimal set of policies.

[Step 2.] According to the socially optimal set of policies of (A34), (A35) and (A36) in the proof of Lemma 3, $\frac{d\tau_K^{s^*}}{dz_T} = 0, \frac{d\tau_L^{s^*}}{dz_T} = 0, \frac{d\mu_L^{s^*}}{dz_T} = 0; \frac{d\tau_K^{s^*}}{dz_K} = 0, \frac{d\tau_L^{s^*}}{dz_K} = 0, \frac{d\mu_L^{s^*}}{dz_K} = 0;$
 $\frac{d\tau_K^{s^*}}{dz_L} = 0, \frac{d\tau_L^{s^*}}{dz_L} = 0$ and $\frac{d\mu_L^{s^*}}{dz_L} = 0$. Therefore, unbiased technological progress (an increase in z_T, z_K or z_L) does not affect the socially optimal set of policies, either.

[Step 3.] Based on the above two steps and (A39), unbiased technological progress (an increase in z_T, z_K or z_L) does not affect the politico-economic output distortion (δ). That is, $\frac{d\delta}{dz_T} = 0, \frac{d\delta}{dz_K} = 0$ and $\frac{d\delta}{dz_L} = 0$.

A9. Proof of Proposition 6

[Step1.] Applying the Implicit Function Theorem to (A31) of the proof of Lemma 3,

$$\frac{d\mu_L^{p^*}}{d\alpha} = 0. \quad (\text{A40})$$

Thus, biased technological progress (an increase or a decrease in α) does not affect the politically optimal degree of labor income tax progressivity. By the same logic, based on (A36) of the proof of Lemma 3,

$$\frac{d\mu_L^{s^*}}{d\alpha} = 0. \quad (\text{A41})$$

Hence, biased technological progress does not affect the socially optimal set degree of labor income tax progressivity, either.

[Step 2.] According to (A39) and the step 1 above,

$$\frac{d\delta}{d\alpha} = 0. \quad (\text{A42})$$

Hence, biased technological progress does not affect the degree of politico-economic output distortion. Notice that each step of this proof holds regardless of whether the condition of (29) is met or not.

Notes

1. Being consistent with Figure 2, studies such as Carmines *et al.* (2012) showed that the US voters were also more polarized in the past decades. Increasing political polarization is also found in other advanced economies such as Canada, UK, Ireland, and the like (e.g., Keivins and Soroka, 2018; Duffy *et al.*, 2019; Draca and Schwarz, 2020).
2. Without specifying utility function, the effect of technological progress on political polarization is not identifiable. For analytical tractability, this paper employs logarithm function of consumption utility which has been widely adopted by various studies (e.g., Benabou, 2002; Corneo, 2002; Guerreiro *et al.*, 2017; Heathcote *et al.*, 2017) and shown to be consistent with empirical findings of labor supply (Chetty, 2006).
3. Some scholars like Acemoglu (2002) modeled technological progress as a change in the price elasticity of substitution between labor and capital. By definition, the price elasticity of substitution between labor and capital measures how *responsive* firms' decisions on the two inputs to a change in the input prices determined in factor markets are. Instead, this paper models technological progress in terms of increases in the parameters of z_T , z_K , z_L and α of the production function (5) which directly raise productivity for any given amounts of inputs, independent of firms' decisions and factor markets.
4. Different from Zhu (1992), Chari *et al.* (1994) and Werning (2007) that analyzed how a social planner adjusts capital and labor income tax rates responding to short-run random technology shocks, this study examines how voters' decisions of capital and labor income tax rates respond to long-run changes in production technology.
5. If we take conventional approach of using Lagrange multiplier(s) for the competitive-equilibrium conditions (i), (ii) and (iii) in obtaining each voter's ideal policy in this market economy, it is inevitable that the formula for each voter's ideal set of policies contains the Lagrange multiplier(s). Thus, without imposing arbitrary assumptions on how the four technology parameters (z_T , z_K , z_L and α) affect the Lagrange multiplier(s), the conventional approach cannot clearly identify the effect of technological progress, if any, on political polarization.
6. As β increases, individual voters put more weights on the future so that they save more by decreasing the value of (19). In turn, more savings lead to more capital wealth of an individual voter, which exerts wealth effect on the individual voter's optimal labor supply, as appears in (17).
7. Moreover, to avoid the dismal case of no provision of public goods, it is ideal for every voter to collect income tax although income taxation incurs efficiency loss. To minimize efficiency loss, first, labor income tax is collected even when capital income tax is not. Only when preference for public goods (χ) is high enough, capital income tax is also collected.
8. As pointed out by many scholars (e.g., Keane, 2011), there is a discrepancy between estimated Frisch labor-supply elasticity (from individual-level data) and calibrated Frisch labor-supply elasticity (from aggregate data), because the latter reflects the non-working population while the former does not. As all individuals are working at a competitive general equilibrium of the model of this paper, estimated Frisch elasticity is considered. Peterman (2016) provided various estimates of Frisch elasticity with individual-level US data. Among them, the most relevant estimate, which is 0.2, is adopted for this study. In fact, 0.2 is equal to the median value of various estimates for Frisch elasticity, according to Keane (2011).
9. In particular, 1979 and 1980 waves of PSID are used. As PSID provides information on taxable income of one year before the survey, these two waves provide income data of 1978 and 1979 for calculating federal and state taxes. PSID provides information for TAXSIM to simulate federal and state income taxes after 1978. Using TAXSIM, pre-tax and post-tax labor incomes of the respondents of PSID are calculated. Then, using the tax function of (8), we run regression of the logarithm of post-tax labor income on the logarithm of pre-tax labor income to estimate the degree of labor income tax progressivity. In this regression, unlike Heathcote *et al.* (2017), we do not include government transfers such as AFDC and Social Security benefit in post-tax labor income; hence, our estimate of the degree of labor income tax progressivity is lower than their estimate of degree of tax progressivity.
10. The DW-NOMINATE score measurement of political polarization in Figure 2 is not directly comparable to our measurement of (28), although the two measurements are positively related to each other. First, (28) is much

more straightforward and relies much less calculation assumptions than the DW-NOMINATE score. Second, unlike the DW-NOMINATE score that is obtained from data of voting not only on taxation policies but also on other policies such as foreign policies, environment policies, immigration policies and the like, (28) bases only on taxation policies.

References

- Acemoglu, D. (2002), "Technical Change, Inequality, and the Labor Market", *Journal of Economic Literature*, 40(1): 7-72.
- Alesina, A. and Tabellini, G. (1990), "A Positive Theory of Fiscal Deficits and Government Debt", *Review of Economic Studies*, 57(3): 403-414.
- Armour, P., Burkhauser, R. and Larrimore, J. (2016), "Using the Pareto Distribution to Improve Estimates of Topcoded Earnings", *Economic Inquiry*, 54(2): 1263-1273.
- Azzimonti, M. and Talbert, M. (2014), "Polarized Business Cycles", *Journal of Monetary Economics*, 67: 47-61.
- Benabou, R. (2000), "Unequal Societies: Income Distribution and the Social Contract", *American Economic Review*, 90(1): 96-129.
- Benabou, R. (2002), "Tax and Education Policy in a Heterogeneous-agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?", *Econometrica*, 70(2): 481-517.
- Carmines, E, Ensley, M. and Wagner, M. (2012), "Who Fits the Left-Right Divide? Partisan Polarization in the American Electorate", *American Behavioral Scientist*, 56(12): 1631-1653.
- Caselli, F. and Feyrer, J. (2007), "The Marginal Product of Capital", *Quarterly Journal of Economics*, 122(2): 535-568.
- Chari, V., Christiano, L. and Kehoe, P. (1994), "Optimal Fiscal Policy in a Business Cycle Model", *Journal of Political Economy*, 102(4): 617-652.
- Chetty, R. (2006), "A New Method of Estimating Risk Aversion", *American Economic Review*, 96(5): 1821-1834.
- Corneo, G. (2002), "The Efficient Side of Progressive Income Taxation", *European Economic Review*, 46(7): 1359-1368.
- Davis, N. and Dunaway, J. (2016), "Party Polarization, Media Choice, and Mass Partisan-ideological Sorting", *Public Opinion Quarterly*, 80(S1): 272-297.
- Draca, M. and Schwarz, C. (2020), "How Polarized are Citizens? Measuring Ideology from the Ground-Up", Mimeo.
- Druckman, J., Peterson, E. and Slothuus, R. (2013), "How Elite Partisan Polarization Affects Public Opinion Formation", *American Political Science Review*, 107(1): 57-79.
- Duca, J. and Saving, J. (2016), "Income Inequality and Political Polarization: Time Series Evidence over Nine Decades", *Review of Income and Wealth*, 62(3): 445-466.
- Duffy, B., Hewlett, K., McCrae, J. and Hall, J. (2019), *Divided Britain?: Polarisation and Fragmentation Trends in the UK*, The Policy Institute at King's College: London.

- Elsby, M., Hobijn, B. and Şahin, A.. (2013), “The Decline of the US Labor Share”, *Brookings Papers on Economic Activity*, 44(2): 1-63.
- Feldstein, M. (1969), “The Effects of Taxation on Risk Taking”, *Journal of Political Economy*, 77(5): 755-764.
- Garand, J. (2010), “Income Inequality, Party Polarization, and Roll-call Voting in the US Senate”, *Journal of Politics*, 72(4): 1109-1128.
- Gentzkow, M. and Shapiro, J. (2011), “Ideological Segregation Online and Offline”, *Quarterly Journal of Economics*, 126(4): 1799-1839.
- Guerreiro, J., Rebelo, S. and Teles, P. (2017), “Should Robots be Taxed?” *NBER Working Paper*, 23806.
- Heathcote, J., Storesletten, K. and Violante, G. (2014), “Consumption and Labor Supply with Partial Insurance: An Analytical Framework”, *American Economic Review*, 104(7): 2075-2126.
- Heathcote, J., Storesletten, K. and Violante, G. (2017), “Optimal Tax Progressivity: An Analytical Framework”, *Quarterly Journal of Economics*, 132(4): 1693-1754.
- Karabarbounis, L. and Neiman, B. (2014), “The Global Decline of the Labor Share”, *Quarterly Journal of Economics*, 129(1): 61-103.
- Keane, M. (2011), “Labor Supply and Taxes: A Survey”, *Journal of Economic Literature*, 49(4): 961-1075.
- Kevins, A. and Soroka, S. (2018), “Growing Apart? Partisan Sorting in Canada, 1992–2015”, *Canadian Journal of Political Science*, 51(1): 103-133.
- Kimball, M. and Shapiro, M. (2008), “Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?”, *NBER Working Paper*, 14208.
- León-Ledesma, M. and Satchi, M. (2019), “Appropriate Technology and Balanced Growth”, *Review of Economic Studies*, 86(2): 807-835.
- Lindqvist, E. and Östling, R. (2010), “Political Polarization and the Size of Government”, *American Political Science Review*, 104(3): 543-565.
- McCarty, N., Poole, K. and Rosenthal, H. (1997), *Income Redistribution and the Realignment of American Politics*, Washington, DC: AEI Press.
- Persson, M. (1983), “The Distribution of Abilities and the Progressive Income Tax”, *Journal of Public Economics*, 22(1): 73-88.
- Persson, T. and Tabellini, G. (2002), *Political Economics: Explaining Economic Policy*, MIT press.
- Peterman, W. (2016), “Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity”, *Economic Inquiry*, 54(1): 100-120.
- Poole, K. and Rosenthal, H. (1997), *Congress: A Political-Economic History of Roll Call Voting*, New York: Oxford University Press.
- Poole, K. (2005), *Spatial Models of Parliamentary Voting*, New York: Cambridge University Press.
- Prior, M. (2013), “Media and Political Polarization”, *Annual Review of Political Science*, 16: 101-127.
- Solt, F. (2016), “The Standardized World Income Inequality Database”, *Social Science Quarterly*, 97, SWIID Version 7.1.

- Werning, I. (2007), “Optimal Fiscal Policy with Redistribution”, *Quarterly Journal of Economics*, 122(3): 925-967.
- Zhu, X. (1992), “Optimal Fiscal Policy in a Stochastic Growth Model”, *Journal of Economic Theory*, 58(2): 250-289.
- Zingher, J. and Flynn, M. (2018), “From on High: The Effect of Elite Polarization on Mass Attitudes and Behaviors, 1972-2012”, *British Journal of Political Science*, 48(1): 23-45.

Resumen

¿Cómo afecta el progreso tecnológico a la polarización política? Para responder a esta pregunta, en una economía en la que la política fiscal se elige por mayoría y los votantes se diferencian por su capacidad de obtener ingresos, se obtiene la política fiscal ideal de cada votante para medir la polarización política como distancia entre las políticas ideales de los votantes de izquierdas y de derechas. El progreso tecnológico exacerba la polarización política cuando está sesgado por el capital al aumentar la productividad relativa del mismo. Por el contrario, el progreso tecnológico no afecta a la polarización política, cuando es imparcial al preservar la productividad relativa de los factores. Tanto el progreso tecnológico sesgado como el no sesgado no afectan a la distorsión político-económica, mientras que sólo el primero afecta a la polarización política.

Palabras clave: progreso tecnológico, polarización política, productividad.

Clasificación JEL: O33, D72, E62.