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The Case for Uniform Commodity Taxation: A Tax Reform Approach*

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Abstract

Generalizing the tax reform approach of Laroque (2005), Kaplow (2006) and Hellwig (2009), we show that for any arbitrary income tax system differential commodity taxation can be Pareto-dominated by a uniform rate with a suitably reformed income tax. Under particular preference specifications, we derive the explicit income tax reforms required in an representative-agent economy and when there is linear progressive, piecewise linear and nonlinear income taxation. Optimal uniform commodity results of Sandmo (1974), Atkinson and Stiglitz (1976) and Deaton (1979) apply with involuntary unemployment and when individuals differ in preferences for leisure and make extensive labour supply decisions.

Keywords: Uniform commodity taxation, Optimal income tax, Piecewise linear income tax.

JEL Classification: H21, H23, H24.

1. Introduction

The choice of a commodity tax structure is one of the oldest issues in optimal tax analysis and has led to some of the most policy-influential results. The seminal optimal tax paper by Ramsey (1927) introduced the famous inverse-elasticity rule, which has been of continuing relevance for the design of excise taxes as well as for public utility pricing. Corlett and Hague (1953) refined the Ramsey analysis by showing that commodity tax rates should be higher for goods most complementary in consumption with leisure. This Corlett-Hague Theorem

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has been used in various contexts to explain how taxes should deviate from uniformity. An example is its role in explaining whether future consumption should be taxed more heavily than current consumption (Atkinson and Sandmo, 1980; Banks and Diamond, 2010).

With the advent of optimal income tax analysis following Mirrlees (1971), Atkinson (1973), Diamond (1980) and Stiglitz (1982), commodity taxation has assumed more of a supporting role. With some exceptions, the emphasis has been on deriving circumstances under which commodity taxes should be uniform.[1] Then, they could either be dispensed with entirely by subsuming them as part of the income tax system, or if separate commodity taxes were maintained, they could be broad-based value-added taxes with uniform rates to minimize costs of collection and compliance. Three key results on the uniformity of commodity taxes were derived by Sandmo (1974, 1976), Atkinson and Stiglitz (1976) and Deaton (1979). Sandmo showed that in a representative-agent setting, optimal Ramsey commodity taxes should be uniform if preferences are weakly separable in goods and leisure and homothetic in goods. The Atkinson-Stiglitz Theorem showed that if preferences are weakly separable in goods and leisure and identical for all persons, optimal commodity taxes should be uniform if nonlinear income taxes are optimal.[2] The Deaton Theorem showed that if the government were restricted to an optimal linear progressive income tax, commodity taxes should be uniform if preferences are weakly separable in goods and leisure and quasi-homothetic in goods. Quasi-homothetic preferences are homothetic to any point, not necessarily the origin. They lead to linear Engel curves for all goods that are the same for all individuals (Gorman, 1961; Deaton and Muellbauer, 1980).

These results have been influential for tax policy purposes, but they are restrictive in some important senses. First, they were derived under the assumption that income taxes are set optimally. Second, they were derived under the assumption that individuals have the same utility functions. Finally, they apply for the extreme cases in which the government can use either fully nonlinear income taxation or linear progressive income taxation. These two cases rule out the form of income taxation governments typically use, which is piecewise linear income taxation.

Recent literature has generalized the Atkinson-Stiglitz and Deaton Theorems to address these restrictions. First, Laroque (2005) and Kaplow (2006) showed that a version of the Atkinson-Stiglitz Theorem applies even if the nonlinear income tax is not optimal. Starting from an arbitrary nonlinear income tax and differential commodity taxes, a Pareto-improving tax reform combining a move to uniform commodity taxation and a reformed nonlinear income tax is feasible if preferences are weakly separable. Mirrlees *et al.* (2011) drew on these results to argue in favour of a VAT with uniform tax rates.[3] Second, Hellwig (2009) proved a parallel generalization of the Deaton Theorem. Starting with differential commodity taxes and an arbitrary linear progressive income tax, a Pareto-improving tax reform involving a move to uniform commodity taxes and a reform of the linear progressive income tax is feasible if preferences are separable in goods and leisure and quasi-homothetic in goods. Recently, Boadway and Cuff (2022) have generalized Deaton (1979) and Hellwig (2009). Both results apply if the income tax is piecewise linear. As well, they apply if individuals have different preferences for leisure, and if incomes are determined under different labour market assumptions, including extensive labour supply choices and involuntary unemployment.

Laroque, Kaplow, Hellwig, and Boadway and Cuff derive their results using a similar methodology. In this paper, we exploit this methodology to synthesize existing results and to extend them to other cases. We show that the types of income tax reforms required to obtain Pareto-improvements when commodity taxes are made uniform take a common form in the various applications. An important advantage of this methodology is that it only involves evaluating Pareto-improving tax reforms and does not require specifying a social welfare function unlike with the standard optimal tax approach. Therefore, our results are of greater generality than the Atkinson-Stiglitz and Deaton Theorems.

2. The Setting

We begin by setting out the general features of the economy. Individuals can differ in some innate characteristics that affects the amount of income they choose to earn. Individual differences may reflect differences in the cost of labour force participation as in the extensive-margin labour supply literature (Diamond, 1980; Saez, 2002), differences in wage rates as in the intensive-margin literature (Mirrlees, 1971), or differences in preferences for leisure (Cuff, 2000; Boadway *et al*., 2000; Choné and Laroque, 2010; Fleurbaey and Maniquet, 2011). Individuals may either participate in the labour market and earn income by supplying labour, or they may be unemployed. The reason for unemployment need not be specified, but it could be voluntary – individuals choose not to work – or involuntary – they are unable to work or unable to find a job. An individual's labour supply decision could depend on both their productivity (wage rate) and their preferences for leisure. Individuals spend their income on a vector of consumption goods. We assume preferences are weakly separable in goods versus both labour and a public good.

More precisely, there are N individuals who participate in the labour market and earn a positive income. Participants are indexed by i such that those with higher i have higher income y, that is, $y^{i+1} > y^i$.[5] Higher incomes can be due to higher productivities (wage rates) or lower preferences for leisure. Those who do not participate in the labour market earn zero income. Such individuals may be unemployed because they are unable to work, choose not to work, or are involuntarily unemployed. We call these individuals collectively *non-participants*.

There are m goods, denoted by x_j for $j = 1, \cdots, m$, which are produced by a linear technology using only labour as an input. Individuals' income is their effective labour supply or the output they produce. Individual preferences can be represented by a strictly quasi-concave utility function that is weakly separable in consumption goods versus income and a pure public good. For participant i, utility is given by $u^i(\phi(x_1, \cdots, x_m), y, g)$,[6] and for any non-participant, utility is $u^0(\phi(x_1, \cdots, x_m), 0, g)$.[7] In the special case of the Mirrlees (1971) model of optimal income taxation where individuals differ only in their wage rate w, utility can be written $u^i(\phi(x_1, \cdots, x_m), y/w, g)$. Importantly, we assume that the sub-utility function $\phi(x_1, \cdots, x_m)$ is identical for all individuals, and in some settings, it is quasi-homothetic or homothetic.

The government imposes an income tax and a set of commodity taxes. We assume that the income tax can take a general nonlinear form, $T(y)$, with $T(0) < 0$ and marginal tax rates

$T'(y)$. Following Saez *et al.* (2012), we define $1 - T'(y)$ as the *net-of-tax rate*. We also consider restricted income tax systems as described further below. Commodity taxes on the m goods are at the ad valorem rates $\boldsymbol{\tau} \equiv (\tau_1, \cdots, \tau_j, \cdots, \tau_m)$. Let consumer prices be $q_j = (1 + \tau_j)p_j$ for $j = 1, \cdots, m$, where p_j are producer prices, and define $\mathbf{q} \equiv (q_1, \cdots, q_m)$ and $\mathbf{p} \equiv (p_1, \cdots, p_m)$ as the vectors of consumer and producer prices, respectively. Since the technology is linear, producer prices are fixed. The budget constraints for non-participants and participants are

$$\sum_{j=1}^{m} q_j x_j^0 = -T(0) \equiv d^0, \quad \sum_{j=1}^{m} q_j x_j^i = y^i - T(y^i) \equiv d^i, \ i = 1, \cdots, N \tag{1}$$

where d^0 and d^i refer to disposable incomes.

Individual outcomes are determined in sequence, with earlier decisions anticipating later outcomes. First, participation in the labour market is determined, either by individual choice or involuntarily. Next, those who decide to participate choose labour supply and therefore earnings on which they pay taxes. Non-participants receive a transfer from the government. Finally, both participants and non-participants choose how to spend their disposable incomes on the m goods. Our analysis is agnostic about how income is determined.

Since utility is weakly separable in goods and the sub-utility function $\phi(\cdot)$ is the same for all persons, we can solve the inner problem for an individual with given disposable income d^h:

$$\max_{\{x_1, \cdots, x_j, \cdots, x_m\}} \phi(x_1, \cdots, x_m) \ \text{ s.t. } \sum_{j=1}^{m} q_j x_j = d^h, \ h = 0,1, \cdots, N \tag{2}$$

where the superscript 'h' includes both participants and non-participants. This yields uncompensated demands $\mathbf{x}(\mathbf{q}, d^h) = (x_1(\mathbf{q}, d^h), \cdots, x_m(\mathbf{q}, d^h))$ and indirect sub-utility

$$v(\mathbf{q}, d^h) = \phi\big(x_1(\mathbf{q}, d^h), \cdots, x_m(\mathbf{q}, d^h)\big). \tag{3}$$

By separability, the demand for goods does not depend on g. An individual's choice of y^h solves:

$$\max_{\{y\}} \ u^h(v(\mathbf{q}, d^h), y, g)$$

where d^h satisfies (1) and includes both participants and non-participants.

For the special case used by Deaton (1979) to study uniform commodity taxation when a linear progressive income tax is used, the sub-utility of goods is quasi-homothetic. In this case, indirect sub-utility in (3) can be written:

$$v(\mathbf{q}, d^h) = \mu(\mathbf{q}) + \psi(\mathbf{q})d^h, \ \ h = 0,1, \cdots, N \tag{4}$$

Using Roy's Theorem, commodity demands can be written as follows:

$$x_j^h = -\frac{\mu_{q_j}(\mathbf{q}) + \psi_{q_j}(\mathbf{q})d^h}{\psi(\mathbf{q})}, \quad \begin{matrix} h = 0,1, \cdots, N \\ j = 1, \cdots, m \end{matrix} \tag{5}$$

Thus, Engel curves are linear in disposable incomes and have the same slopes and intercepts for all individuals. If the sub-utility of goods is homothetic, then $\mu(\cdot) = 0$, so indirect sub-utility and commodity demands become

$$v(\mathbf{q}, d^h) = \psi(\mathbf{q}) d^h, \;\; x_j^h = -\frac{\psi_{q_j}(\mathbf{q})}{\psi(\mathbf{q})} d^h, \;\; \begin{matrix} h = 0,1,\cdots,N \\ j = 1,\cdots,m \end{matrix} \tag{6}$$

We will refer to these special cases in what follows.

Our analysis takes the form of a tax reform. We begin with a *status quo* policy situation in which the government imposes an arbitrary income tax system $T(y)$, a set of differential commodity taxes $\boldsymbol{\tau}$, with $\tau_l \neq \tau_k$ for at least one combination of l, k, and a public good g. We denote the status quo choices of goods and income for a type-h individual as $\mathbf{x}^h(s)$ and $y^h \geq 0$. The value function for the sub-utility problem (2) is denoted

$$\omega^h(s) \equiv v(\mathbf{q}, d^h), \;\; h = 0,1,\cdots,N \tag{7}$$

Then, for various forms of $T(y)$, we show conditions under which the status quo allocation can be Pareto-dominated by moving to uniform commodity taxes and a suitably reformed income tax. We derive and interpret the income tax reforms that must accompany the move to uniform commodity taxes. We consider various cases which have their parallels in the literature: identical individuals (characterized by a representative individual), and heterogeneous households facing i) a linear progressive income tax, ii) a piecewise linear income tax and iii) a nonlinear income tax. The results we present have their counterparts in Sandmo (1976), Atkinson and Stiglitz (1976), Deaton (1979), and Boadway and Cuff (2022).

Our method of tax reform analysis is similar in all cases and is a refinement of that initiated by Laroque (2005) and Kaplow (2006) for the Mirrlees optimal nonlinear income tax setting, and extended to linear progressive taxes by Hellwig (2009) and piecewise linear income taxes by Boadway and Cuff (2022). It consists of three steps. First, the differential commodity taxes are replaced by a uniform commodity tax system holding sub-utility constant. Second, the income tax is reformed such that all individuals can – and will – choose the status quo income level and achieve the status quo sub-utility of goods. Third, it is shown that the government has surplus tax revenue from the above tax reforms and can use it to achieve a Pareto improvement.

3. Identical Individuals

This case is the counterpart to the Corlett and Hague (1953) analysis. A representative taxpayer supplies labour and consumes two goods, and initially faces a uniform commodity tax system. Lump-sum taxes are ruled out since they would dominate. They derive conditions under which commodity taxes should deviate from uniformity with tax revenue held constant. They obtain the Corlett-Hague Theorem: the commodity tax rate should be increased on the good which is more complementary with leisure and decreased on the other. Sandmo (1976) showed that if preferences are homothetic, taxes should not deviate from uniformity. Our

approach is to begin with differentiated commodity taxes and an arbitrary proportional income tax and show that when preferences are homothetic, tax reform should move to uniform commodity taxes. This representative taxpayer case (or more properly the case of many identical individuals) serves as an introduction to the tax reform approach that we apply to other cases.

The status quo problem for the representative individual is:

$$\max_{\{x,y\}} u\big(\phi\big(x_1,\cdots,x_j,\cdots,x_m\big),y,g\big) \text{ s.t. } \sum_{j=1}^{m} q_j\, x_j = (1-t)y \tag{8}$$

where t is the proportional income tax and $q_j = (1+\tau_j)p_j$, with $\tau_l \neq \tau_k$ for at least some l,k. Here, the net-of-tax rate, $1-t$, is the same for all individuals and $(1-t)y$ is disposable income.

Given separable preferences in consumption goods, the representative individual with disposable income $(1-t)y$ chooses $\mathbf{x}$ that solves the same inner problem (2) as above. This yields uncompensated demands $\mathbf{x}(\mathbf{q},(1-t)y)$ and indirect sub-utility $v(\mathbf{q},(1-t)y)$. We assume homothetic preferences over goods, so indirect sub-utility and commodity demands satisfy (6). Optimal demands are linear and go through the origin. Relative commodity demands will therefore be independent of income.

The representative individual's choice of y then solves:

$$\max_{\{y\}} u(\psi(\mathbf{q})(1-t)y, y, g).$$

As above, the solution to the status quo problem is denoted by $x_j(s), j=1,\cdots,m$, and $y(s)$ with indirect sub-utility (from (6) and (7))

$$\omega(s) = \psi(\mathbf{q})(1-t)y \tag{9}$$

for all levels of disposable income $(1-t)y$.

We show, following Sandmo (1976),[8] that we can achieve a Pareto improvement over the status quo by moving to an arbitrary uniform commodity tax ($\tau_j = \tau$ for all j) and an alternate proportional income tax $\hat{t}$ which will depend on τ. The combination of τ and $\hat{t}$ is indeterminant.

Proposition 1. *Identical Individuals (Sandmo, 1974 and 1976).*

Assume the utility function for the representative individual takes the form $u(\phi(\mathbf{x}),y,g)$ *where sub-utility* $\phi(\mathbf{x})$ *is homothetic. Let* $(\mathbf{x}(s),y(s),g)$ *be the allocation under an arbitrary proportional income tax* t*, with differential commodity taxes* $\boldsymbol{\tau}$*. Another allocation can be implemented by a uniform commodity tax* τ *and a proportional change in the net-of-tax rate that gives the same utilities as the allocation* $(\mathbf{x}(s),y(s),g)$ *and generates more government tax revenues. The additional revenues can be used to make a strict Pareto improvement.*

As mentioned, the proof proceeds in three steps. First, differential commodity taxes are replaced by a uniform commodity tax holding sub-utility constant. Second, the proportional in-

come tax is changed so that the representative individual can attain the status quo level of utility under the uniform commodity tax system. Third, we show that the government obtains additional tax revenue from the first two steps and can use it to make a Pareto improvement. Note that when the income tax is proportional, a change in the uniform commodity tax rate will have the same effect on the individual's budget constraint as a change in the proportional income tax rate. Consequently, our proof can also be applied by eliminating all differential commodity taxes and then adjusting the uniform commodity tax rate to achieve a Pareto improvement.

Step 1. *Replacement of differential commodity taxes.*

Suppose the government replaces the differential commodity tax system with a uniform commodity tax at the rate τ. Consider the following problem which minimizes consumption expenditure under the uniform commodity tax subject to attaining the same level of sub-utility as in the original allocation/tax system.

$$\min_{\{x_j\}} \sum_j (1+\tau) p_j x_j \quad \text{s.t. } \phi(x_1, \cdots, x_m) = \omega(s)$$

where $\omega(s)$ satisfies (9). The solution is

$$\hat{x}\big((1+\tau)\mathbf{p}, \omega(s)\big) = \Big(\hat{x}_1\big((1+\tau)\mathbf{p}, \omega(s)\big), \cdots, \hat{x}_m\big((1+\tau)\mathbf{p}, \omega(s)\big)\Big),$$

and minimized total expenditures satisfy

$$e\big((1+\tau)\mathbf{p}, \omega(s)\big) = \sum_j (1+\tau) p_j \hat{x}_j\big((1+\tau)\mathbf{p}, \omega(s)\big) < \sum_j (1+\tau) p_j x_j \tag{10}$$

for any other commodity choices x_j satisfying $\phi(x_1, \cdots, x_m) = \omega(s)$. This implies

$$\sum_j p_j \hat{x}_j\big((1+\tau)\mathbf{p}, \omega(s)\big) < \sum_j p_j x_j(s) \tag{11}$$

where $x_j(s), j = 1, \cdots, m$ are the optimal demands in the status quo. That is, the commodity bundle $\hat{\mathbf{x}}((1+\tau)\mathbf{p}, \omega(s))$ takes less resources to produce than the status quo bundle $\mathbf{x}(\mathbf{q}, (1-t)y)$, and both bundles achieve the same level of sub-utility $\omega(s)$.

Step 2. *Adjustment of the income tax.*

By duality of the individual's inner problem, (2), we have the following relationship under the uniform commodity tax

$$v\Big((1+\tau)\mathbf{p}, e\big((1+\tau)\mathbf{p}, \omega(s)\big)\Big) = \omega(s)$$

since the expenditure function satisfies $e((1+\tau)\mathbf{p}, \omega(s)) = (1-t)y$. Given that sub-utility $\phi(\cdot)$ is homothetic, we can rewrite this using (6) as follows:

$$\psi\big((1+\tau)\mathbf{p}\big) \cdot e\big((1+\tau)\mathbf{p}, \omega(s)\big) = \omega(s) = \psi(\mathbf{q})(1-t)y$$

where the last equality follows from (9). Therefore, the minimized expenditure needed when there is a uniform commodity tax to achieve the same sub-utility as in the status quo is:

$$e\big((1+\tau)\mathbf{p}, \omega(s)\big) = \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)}(1-t)y. \tag{12}$$

Suppose the reformed income tax satisfies

$$1-\hat{t} = \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)}(1-t). \tag{13}$$

Using (13) and the definition of $e((1+\tau)\mathbf{p}, \omega(s))$ from (10), we can rewrite (12) as

$$\sum_j (1+\tau)p_j\,\hat{x}_j = (1-\hat{t})y. \tag{14}$$

After the tax reform, the representative individual with income y can purchase the bundle of goods $\hat{\mathbf{x}}$ with a uniform commodity tax and achieve the same level of sub-utility as in the status quo. The individual's choice of income in the adjusted tax system is given by

$$\max_{\{y\}} u\big(v\big((1+\tau)\mathbf{p}, (1-\hat{t})y\big), y, g\big).$$

Since by construction, we have for any y

$$v(\mathbf{q}, (1-t)y) = v\big((1+\tau)\mathbf{p}, (1-\hat{t})y\big),$$

the representative individual will choose y and be just as well off as in the status quo.

The form of the income tax rate change in (13) is intuitive and reappears in the other cases considered below. The net-of-tax rate terms $1-t$ and $1-\hat{t}$ are the slopes of the individual's budget line. They represent the rates at which income can be converted to consumption and reflect the size of the labour-consumption distortion. The change in the net-of-tax rate $1-t$ in Step 2 required to compensate for the change in commodity taxation in Step 1 is given by $\psi(\mathbf{q})/\psi((1+\tau)\mathbf{p})$ in (13). This is an index of the difference in consumer prices before and after the commodity tax reform.

Step 3. *Effect on tax revenues.*

The government budget constraint – or equivalently, the economy's resource constraint– before and after the tax reform can be written:

$$p_g g = y - \sum_j p_j\,x_j(s) < y - \sum_j p_j\,\hat{x}_j$$

where p_g is the per unit cost of g, y is total output, $\sum_j p_j x_j$ is total consumption, and the inequality follows from (11). Therefore, the government has extra tax revenue that can be used to increase the public good and achieve a Pareto improvement.

Proposition 1 holds for any arbitrary income tax, including the optimal proportional tax. Therefore, we have shown that with identical individuals who have preferences that are weakly separable in goods versus leisure and a public good, and are homothetic in goods, the optimal commodity tax rate is uniform (Sandmo, 1976). With homothetic preferences for goods, the needed adjustment to the income tax is a proportional change in the net-of-tax rate in Step 2. This instead could be achieved by adjusting the uniform commodity tax rate after eliminating the differential taxes and leaving the income tax rate unchanged.[9]

4. Linear progressive income taxation

Now suppose individuals are heterogeneous and their differences give rise to differences in income. They could differ in productivity as in the standard Mirrleesian model (Mirrlees, 1971), or they could differ in preferences for leisure (Cuff, 2000; Boadway *et al.*, 2000; Choné and Laroque, 2010; Fleurbaey and Maniquet, 2011). They may freely choose their incomes by varying their labour supply along the intensive or extensive margin (Diamond, 1980; Saez, 2002), or their choices may be constrained by the possibility of involuntary unemployment due to search frictions or efficiency wages (Hungerbühler *et al*., 2006; Lehmann *et al*., 2011; Boadway and Cuff, 2014; Jacquet *et al*., 2014; Kroft *et al*., 2020) or a minimum wage (Hungerbühler and Lehmann, 2009). Our analysis applies whatever mechanisms determine labour incomes.

In this section, we assume that government tax instruments are restricted to commodity taxes $\boldsymbol{\tau}$ and a linear progressive income tax with a marginal tax rate t and a lump-sum transfer a. The net-of-tax rate $1-t$ is again the same for all individuals. We begin with a status quo allocation under an arbitrary linear progressive tax and differential commodity taxes, and we consider circumstances under which the move to uniform commodity taxes accompanied by a reform of the income tax can be Pareto-improving. We draw on Deaton (1979), who showed that when preferences are quasi-homothetic in goods and the income tax is constrained to be linear progressive, commodity taxes will be uniform in the optimum. The analysis we present generalizes that of Hellwig (2009).

Although we are dealing with an economy consisting of heterogeneous individuals, there is no need to specify a social welfare function as we consider only Pareto-improving tax reforms. This is advantageous because we can avoid the conceptually difficult task of specifying social weights when individuals differ in preferences for leisure (Cuff, 2000; Boadway *et al*., 2000; Fleurbaey and Maniquet, 2011). Nonetheless, the results we obtain on Pareto-improving tax reforms allow us to infer as a corollary that if taxes were set optimally using a social welfare function, uniform commodity taxes would be optimal.

Utility functions for participants and non-participants are $u^i(\phi(x_1,\cdots,x_m),y,g)$ and $u^0(\phi(x_1,\cdots,x_m),0,g)$ as above, where sub-utility functions $\phi(\mathbf{x})$ are identical for all individuals but overall utility functions can differ because of different preferences for leisure (or possibly for the public good). Participants' incomes y^i can be determined from different models of labour markets as mentioned, and we assume individuals are indexed such that

$y^{i+1} > y^i$. Our assumption that there is only one individual who chooses any given income level is without loss of generality. Having an arbitrary number of individuals choosing any given level of income would not affect the results. The budget constraints (1) for non-participants and participants are:

$$\sum_j q_j x_j^0 = a \equiv d^0, \ \sum_j q_j x_j^i = (1-t)y^i + a \equiv d^i, \ i = 1, \cdots, N \tag{15}$$

Individual outcomes are determined in sequence, with earlier decisions anticipating later outcomes. First, participation in the labour market is determined, either by individual choice or involuntarily. Then, those who decide to participate choose labour supply and therefore earnings on which they pay taxes. Non-participants receive a transfer from the government. Finally, both participants and non-participants choose how to spend their disposable incomes on the m goods.

With separable preferences in consumption goods, an individual with disposable income d^h chooses $\mathbf{x}$ that solves the same inner problem (2) as above. This yields uncompensated demands $\mathbf{x}(\mathbf{q}, d^h)$ and indirect sub-utility $v(\mathbf{q}, d^h)$ where $h = 0, 1, \cdots, N$ includes both participants and non-participants. We assume quasi-homothetic preferences over goods, so indirect sub-utility satisfies (4) and demands for goods are given by (5).

In our initial – or status quo – policy setting, the government chooses an arbitrary linear progressive income tax system with parameters (a, t) and differential commodity taxes $\boldsymbol{\tau}$. The allocations chosen in the status quo are $y^i(s)$ and $\mathbf{x}^i(s)$ for participants $i = 1, \cdots, N$, and $\mathbf{x}^0(s)$ for non-participants. The values of the sub-utility functions in the status quo are:

$$\omega^0(s) \equiv \phi\big(\mathbf{x}^0(s)\big); \ \omega^i(s) \equiv \phi\left(\mathbf{x}^i(s)\right), \ i = 1, \cdots, N \tag{16}$$

Proposition 2. *Linear Progressive Tax System (Deaton, 1979).*

Assume utility functions for participants and non-participants take the forms $u^i(\phi(\mathbf{x}), y, g)$ *and* $u^0(\phi(\mathbf{x}), 0, g)$ *where sub-utility* $\phi(\mathbf{x})$ *is quasi-homothetic. Let* $(\mathbf{x}^h(s), y^h(s), g)$ *be the allocation of individual* h *for* $h = 0, 1, \cdots, N$ *under an arbitrary linear progressive income tax system* (a, t) *with differential commodity taxes* $\boldsymbol{\tau}$. *Another allocation can be implemented by a uniform commodity tax rate* τ, *a proportionate change in the net-of-tax rate and a change in the lump-sum transfer that gives the same utilities as the allocation* $(\mathbf{x}^h(s), y^h(s), g)$ *and generates more government tax revenues which can be used to make a strict Pareto improvement.*

The proof of Proposition 2 follows the same three steps as Proposition 1. However, there are now $N+1$ participants and non-participants, and the three steps apply to each type-h. Step 1, which involves replacing differential with uniform commodity taxes and solves for the commodity demands that yield the status quo sub-utility, is virtually identical. Eqs. (10) and (11) for the expenditure function and costs of production become:

$$e\big((1+\tau)\mathbf{p}, \omega^h(s)\big) = \sum_j (1+\tau) p_j \hat{x}_j^h\big((1+\tau)\mathbf{p}, \omega^h(s)\big). \tag{17}$$

and

$$\sum_j p_j \hat{x}_j^h\big((1+\tau)\boldsymbol{p}, \omega^h(s)\big) < \sum_j p_j x_j^h(s) \tag{18}$$

Fewer resources are required to produce $\hat{\mathbf{x}}^h((1+\tau)p, \omega^h(s))$ than to produce the status quo allocation $\mathbf{x}^h(s)$ when both commodity bundles yield the same sub-utility.

Step 2. *Adjustment of the linear progressive income tax.*

For each individual h, the Step 1 compensated demands $\hat{\mathbf{x}}^h((1+\tau)p, \omega^h(s))$ – which yields the same sub-utility as the status quo allocation – combined with the status quo income $y^h(s)$ yields the same level of utility as in the status quo. We now show that this allocation can be implemented for all individuals with uniform commodity taxes, τ, and a reformed linear progressive income tax $(\hat{a}, \hat{t})$.

Since sub-utility $\phi(\mathbf{x})$ is the same in the Step 1 problem as in the status quo, we can write (4) for the Step 1 problem with uniform commodity taxes as:

$$\omega^h(s) = \mu\big((1+\tau)\mathbf{p}\big) + \psi\big((1+\tau)\mathbf{p}\big)e\big((1+\tau)\mathbf{p}, \omega^h(s)\big),\ h = 0, 1, \cdots, N \tag{19}$$

where $e((1+\tau)\mathbf{p}, \omega^h(s))$, defined by (17), is the amount of expenditures required to finance the Step 1 bundle of goods $\hat{\mathbf{x}}^h$. Using (4) for $\omega^h(s)$ and (17) for $e((1+\tau)\mathbf{p}, \omega^h(s))$, we obtain:

$$\sum_j (1+\tau)p_j \hat{x}_j^h = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} d^h,\ h = 0, 1, \cdots, N \tag{20}$$

Substituting for non-participant and participants' disposable incomes d^h from (15) yields:

$$\sum_j (1+\tau)p_j \hat{x}_j^0 = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} a \tag{21}$$

$$\sum_j (1+\tau)p_j \hat{x}_j^i = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} [(1-t)y^i + a],\ i = 1, \cdots, N \tag{22}$$

Consider an alternative income tax system $(\hat{a}, \hat{t})$ and suppose it is related to the original tax system (a, t) as follows:

$$\hat{a} = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} a \tag{23}$$

$$1 - \hat{t} = \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} (1-t) \tag{24}$$

Substituting (23) and (24) into (21) and (22) gives:

$$\sum_j (1+\tau) p_j \hat{x}_j^0 = \hat{a}, \tag{25}$$

$$\sum_j (1+\tau) p_j \hat{x}_j^i = (1-\hat{t}) y^i + \hat{a}, \; i = 1, \cdots, N, \tag{26}$$

These are the same budget constraints as in (15) with the new tax system and uniform commodity taxes. By construction, (25) and (26) show that each individual h faced with the new income tax system and uniform commodity taxes can choose the original income level $y^h(s)$ and the goods in the post-reform situation $\hat{\mathbf{x}}^h$ that minimize production costs and yield the status quo sub-utility, $\omega^h(s)$. Further, since individual utility depends only on the value of the sub-utility of goods individuals will make the same participation and income choices as in the status quo. Therefore, they will obtain the same utility level as in the status quo.

As Step 1 indicates, if individuals choose the bundle $\hat{\mathbf{x}}^h$, fewer resources are required and they are as well-off as in the original tax system. This means that the government has extra resource available that could be used to increase social welfare in Step 3. The government budget constraint, which is equivalent to the economy's aggregate resource constraint, can be written as follows in the initial and final situations:[10]

$$p_g g = \sum_h y^h(s) - \sum_h \sum_j p_j x_j^h(s) < \sum_h y^h(s) - \sum_h \sum_j p_j \hat{x}_j^h \tag{27}$$

where the inequality follows from (18).

The above proof is based on a revealed preference argument and does not require differentiability in income. Step 1 applies whatever incomes taxpayers choose in the initial allocation, including if they choose not to work or are unable to work. Step 2 then shows that the incomes chosen in the initial allocation can still be supported after differential commodity taxes are made uniform and the income tax suitably reformed. All individuals choose the pre-reform income level and the bundle of goods yielding the pre-reform sub-utility level. Step 3 shows that the government has additional tax revenues that it can use to achieve a Pareto improvement.

The intuition is as follows: The replacement of the differential commodity taxes with a uniform rate results in a change in the relative price of goods versus labour and a change in total commodity taxes paid. These two effects are offset by changes in the net-of-tax rate and in the lump-sum transfer. In particular, by (24), $1 - t$ changes in proportion resulting in a rotation of the budget line. The size and direction of the rotation depends on goods prices in the status quo relative to those after the reform, reflected by $\psi(\mathbf{q})/\psi((1+\tau)\mathbf{p})$ which depends on the size of τ and is analogous to (13) in the identical individuals' case. The change in a in (23) compensates for the change in commodity tax revenues which shifts the intercept of the budget set.

As shown in (5), with quasi-homothetic preferences over goods a change in the prices of goods affects the demand for goods through changes in both $\mu(\cdot)$ and $\psi(\cdot)$. A change in

the commodity prices will both rotate and shift Engel curves for goods. Only the latter effect occurs with homothetic preferences. To ensure individuals can achieve the status quo level of sub-utility with the tax reform when preferences over goods are quasi-homothetic, both net-of-tax rate and the lump-sum transfer must adjust proportionately by the index of the difference in commodity prices before and after the commodity tax reform. The lump-sum component of the tax system must also adjust to compensate for the shift in demands as given by the first term in (23). In the identical individual case, the tax system is proportional and there is no lump-sum component to make this latter adjustment. Thus, when individuals are identical the optimality of uniform commodity taxes holds only in the case of homothetic preferences over goods.

We have demonstrated that the Deaton Theorem – uniform commodity taxation is optimal when preferences over goods are quasi-homothetic – holds when individuals make both extensive and intensive labour supply choices, some individuals may be unemployed and individuals may differ in their preferences for leisure and possibly for the public good. As homothetic preferences are a special case of quasi-homothetic preferences, that is, (7) can be obtained from (4) by setting $\mu(\mathbf{q}) = 0$, Proposition 2 also holds with homothetic preferences. In this case, it follows from (20) with $\mu(\mathbf{q}) = 0$ that only a proportional adjustment in disposable income is required in the tax reform. This can be achieved by adjusting the uniform commodity tax rate rather than adjusting the linear progressive income tax system.[11]

5. Piecewise linear income taxation

The linear progressive income tax is simplest form of progressive income tax and has been widely used in the literature (e. g., Atkinson, 1973; Sheshinski, 1972; Meltzer and Richard, 1981). It is however restrictive and does not correspond with tax structures typically used. Most governments use piecewise linear income taxes which combine a lump-sum component with multiple tax brackets. There has been relatively little analysis of the optimal tax properties of piecewise linear income taxation, largely because results tend to be ambiguous. Some examples include Strawczynski (1988), Sheshinski (1989), Slemrod *et al*. (1994), Apps *et al*. (2011) and Bastani *et al*. (2019). In this section, we draw on Boadway and Cuff (2022) to show how the results of Deaton (1979) and Hellwig (2009) can be generalized to piecewise linear income taxation using the tax reform analysis outlined above. If preferences are quasi-homothetic, an arbitrary piecewise linear income tax combined with differential commodity taxation is Pareto-dominated by a uniform commodity tax and a suitably reformed income tax. The income tax reform involves equal proportional changes in the net-of-tax rate for all individuals.

Consider the following piecewise linear income tax system that characterizes in a simplified form many real world income tax systems. There are an arbitrary number of income tax brackets denoted by $b = 1, \cdots, B$ with corresponding tax rates t_b on income $y \in (\bar{y}_{b-1}, \bar{y}_b]$ where $\bar{y}_0 = 0$. The government chooses the tax rates $\mathbf{t} \equiv (t_1, \cdots, t_B)$, the income bracket thresholds $\bar{\mathbf{y}} \equiv (\bar{y}_1, \cdots, \bar{y}_b)$, and a lump-sum transfer a paid to all individuals regardless of

whether they are working. The tax liability of non-participants is $-a$. Participants with income $y \leq \bar{y}_1$ pay taxes of $t_1 y - a$ while those with $y > \bar{y}_1$ in income tax bracket b pay taxes of:

$$\sum_{k=1}^{b-1} t_k(\bar{y}_k - \bar{y}_{k-1}) + t_b(y - \bar{y}_{b-1}) - a = t_b y - \sum_{k=1}^{b-1}(t_{k+1} - t_k)\bar{y}_k - a, \;\; b = 2, \cdots, B \quad (28)$$

where we have used $\bar{y}_0 = 0$ to obtain the righthand side. Using (28), individual disposable incomes are:

$$\begin{gathered} d^0 = a, \;\; d^{1i} = (1 - t_1)y^{1i} + a, \\ d^{bi} = (1 - t_b)y^{bi} + \sum_{k=1}^{b-1}(t_{k+1} - t_k)\bar{y}_k + a, \;\; b = 2, \cdots, B \end{gathered} \quad (29)$$

where earned incomes are given by y^{1i} and y^{bi} for $b = 2, \cdots, B$. We continue to define the net-of-tax rate as one minus the marginal tax rate on the last dollar of income. With a piecewise linear income tax, the net-of-tax rate varies across the different income brackets.

Individuals now differ both by their participation status, i. e., 0 or i, and by the income bracket where their income y^i is located, i. e., $b = 1, \cdots, B$. To capture this, we use $h = \{0, 1i, bi\}$ to refer respectively to non-participants, participant i in the first income tax bracket and participant i in income tax bracket $b = 2, \cdots, B$. The population of participating individuals with incomes $y^i, i = 1, \cdots, N$ are allocated among the B tax brackets. A given tax bracket will include persons of different incomes, with higher tax brackets containing higher-income persons.

In the status quo policy setting, the government chooses an arbitrary piecewise linear progressive income tax system with parameters $(a, \mathbf{t}, \bar{\mathbf{y}})$, and differential commodity taxes $\boldsymbol{\tau}$. Incomes and consumption vectors chosen in the status quo are again denoted by $y^h(s)$ and $\mathbf{x}^h(s)$, and the value of the sub-utility function for a type-h individual in the status quo is $\omega^h(s) \equiv \phi(\mathbf{x}^h(s))$. We assume that the sub-utility function is quasi-homothetic so the indirect sub-utility function satisfies (4). Denote $d^h(s)$ as the disposable income in the status quo. The status quo budget constraint of individual $h = \{0, 1i, bi\}$ for $b = 2, \cdots, B$ and $i = 1, \cdots, N$ is:

$$\sum_j q_j \, x_j^h(s) = d^h(s). \quad (30)$$

Proposition 3. *Piecewise Linear Income Tax System (Boadway and Cuff, 2022).*

Assume utility functions for participants and non-participants take the forms $u^i(\phi(\mathbf{x}), y, g)$ *and* $u^0((\mathbf{x}), 0, g)$ *where sub-utility* $\phi(\mathbf{x})$ *is quasi-homothetic. Let* $(\mathbf{x}^h(s), y^h(s), g)$ *be the allocation of individual* h *for* $h = 0, 1i, bi$ *under an arbitrary piecewise linear income tax system* $(a, \mathbf{t}, \bar{\mathbf{y}})$*, with differential commodity taxes* $\boldsymbol{\tau}$*. Another allocation can be implemented by a uniform commodity tax* τ*, a common proportional change in net-of-tax rates for all tax brackets holding* $\bar{\mathbf{y}}$ *constant, and a change in the lump-sum transfer that gives the same utilities as the allocation* $(\mathbf{x}^h(s), y^h(s), g)$ *and generates more government tax revenues which can be used to make a strict Pareto improvement.*

The proof of Proposition 3 follows the same three steps as Proposition 2. Step 1 is in fact identical, so both (17) and (18) hold. Fewer resources are required to produce the vector of goods chosen by individuals that achieves the status quo level of sub-utility when the differential commodity taxes are replaced with a uniform rate than the status quo vector of goods. The next step is to show that the allocation in which individuals choose this less resource-intensive vector of goods and their status quo incomes can be achieved with uniform commodity taxes, τ, and a reformed piecewise linear income tax $(\hat{a}, \hat{\mathbf{t}})$. There is no need to adjust income tax brackets.

Step 2. *Adjustment of the piecewise linear income tax.*

As in the linear progressive tax case above, we can write the value of the sub-utility function for the Step 1 problem as (19). Using expression (17) for the expenditure function, (19) becomes:

$$\sum_j (1+\tau)p_j\, x_j^h = \frac{\omega^h(s) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)}$$

Using (4) for the value of indirect utility $\omega^h(s)$ in the status quo situation, this becomes

$$\sum_j (1+\tau)p_j\, x_j^h = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} d^h(s), \quad \begin{matrix} h = \{0, 1i, bi\} \\ b = 2, \cdots, B \end{matrix} \tag{31}$$

where the status quo disposable incomes $d^h(s)$ are given by (29) given status quo incomes $y^h(s)$ and the original income tax system $(a, \mathbf{t})$.

Consider an alternative income tax system $(\hat{a}, \hat{\mathbf{t}})$, and suppose it is related to the original tax system $(a, \mathbf{t})$ as follows:

$$\hat{a} = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} a \tag{32}$$

$$(1 - \hat{t}_b) = \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} (1 - t_b) \quad \text{for} \quad b = 1, \cdots, B \tag{33}$$

where $\bar{\mathbf{y}}$ remains unchanged. For any two income tax brackets b and $b+1$, (33) implies:

$$\hat{t}_{b+1} - \hat{t}_b = \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)} (t_{b+1} - t_b). \tag{34}$$

Substituting (32)-(34) into (31) gives:

$$\sum_j (1+\tau)p_j\, \hat{x}_j^0 = \hat{a}, \tag{35}$$

$$\sum_j (1+\tau)p_j \hat{x}_j^{1i} = (1 - \hat{t}_1) y^{1i}(s) + \hat{a}, \tag{36}$$

$$\sum_j (1+\tau)p_j\hat{x}_j^{bi} = (1-\hat{t}_b)y^{bi}(s) + \sum_{k=1}^{b-1}(\hat{t}_{k+1}-\hat{t}_k)\bar{y}_k + \hat{a},\ b = 2,\cdots,B. \qquad (37)$$

By construction, (35)-(37) show that each individual h faced with the new income tax system and uniform commodity taxes can choose the original income level $y^h(s)$ and the goods in the post-reform situation $\hat{\mathbf{x}}^h$ that minimize production costs and yield the status quo level of sub-utility, $\omega^h(s)$. Therefore, they can obtain the same utility level as in the status quo.

The intuition of this income tax reform is analogous to the linear progressive income tax case. In Step 1, differential commodity taxes are eliminated and replaced by a uniform commodity tax τ. This will have two effects: a change in the relative price of goods versus labour and a change in total commodity taxes paid. These two effects are offset by changes in the net-of-tax rates for each income bracket and in the lump-sum transfer. In particular, by (33), $1-t_b$ changes in the same proportion for all tax brackets resulting in a rotation of the budget line. The size and direction of the rotation depends on goods prices in the status quo relative to those after the reform, reflected by $\psi(\mathbf{q})/\psi((1+\tau)\mathbf{p})$ which depends on the size of τ. This is analogous to the linear progressive income tax case where by (24) there is a proportionate change in $1-t$ which also depends on a similar index of the change in consumer prices. The change in a in (32) shifts the intercept of the budget set and compensates for the change in commodity tax revenues.

In fact, individuals of type h will choose the allocation $(\hat{\mathbf{x}}^h((1+\tau)\mathbf{p}, \omega^h(s)), y^h(s))$. The budget constraints (35)-(37) apply for any income level. By Step 1, when income is y the sub-utility of consumption obtained from the bundle $\hat{\mathbf{x}}^h((1+\tau)p, \omega^h(s))$ is the same as the sub-utility $\omega^h(s)$ obtained under the status quo bundle $\mathbf{x}^h(s)$. Therefore, each point on the budget constraints (35)-(37) give the same combination of y and $\phi(\mathbf{x}^h)$ as the corresponding point on the original budget constraints (30). Equivalently, each value of y yields the same value of utility to an individual under the status quo and new tax systems. The implication is that all individuals have the same opportunities, and therefore will choose the same y after the tax is reformed and will achieve the same level of utility.

As in the linear progressive income tax case, the aggregate resource constraints in the status quo and reformed situations satisfy (27). These are equivalent to the government budget constraints. Therefore, the government has more revenue after the tax reform and can make a Pareto-improvement by increasing the lump-sum transfer a or the provision of public goods g.

6. Nonlinear income taxation

Now suppose we do not restrict the income tax schedule to be piecewise linear and instead allow for any arbitrary nonlinear income tax system. Atkinson and Stiglitz (1976) show that with the optimal nonlinear income tax system in place there is no role for differentiated commodity taxation if preferences are weakly separable in goods versus leisure and identical among all individuals. Uniform commodity taxation is optimal with the rate of commodity

taxation being indeterminate. While this result requires preferences to be weakly separable in goods versus labour, it does not require any further structure on the sub-utility over the set of goods. Laroque (2005) and Kaplow (2006) demonstrate that when preferences are weakly separable in goods and leisure there is no role for differential commodity taxes even in the presence of a non-optimal nonlinear income tax system. A Pareto improvement can be achieved by replacing differential commodity taxes with a uniform commodity tax at an arbitrary rate and adjusting the nonlinear income tax system.

We begin by replicating the Laroque-Kaplow analysis using our three-step approach. We derive an explicit expression for the change in the nonlinear income tax schedule required to generate a Pareto-improvement when uniform commodity taxes are adopted. As we have not placed any restrictions on the sub-utility function $\phi(\mathbf{x})$, the form of the changes in the income tax are fairly general. We then consider the special case of quasi-homothetic sub-utility of goods and show that the income tax reform required is analogous to that obtained in Propositions 2 and 3 above. That is, the net-of-tax rate should change in the same proportion for all income levels.

Utility functions for participants and non-participants take the forms $u^i(\phi(\mathbf{x}), y, g)$ and $u^0(\phi(\mathbf{x}), 0, g)$ where to begin with we impose no restrictions on $\phi(\mathbf{x})$. Under a nonlinear income tax system $T(y)$, the budget constraint for each participant is:

$$\sum_j q_j x_j^i = y^i - T(y^i) \equiv d^i \quad \text{for } i = 1, \cdots, N \tag{38}$$

and for non-participants is

$$\sum_j q_j x_j^0 = -T(0) \equiv d^0 \tag{39}$$

where $q_j = (1 + \tau_j)p_j$. Disposable incomes are d^h where $h = 0$ denotes a non-participant and $h = i$ denotes a type-i participant. We again denote individual choices and level of sub-utility in the status quo using s. The government replaces the differential commodity taxes with a uniform commodity tax at the rate τ and replaces the nonlinear income tax $T(y)$ with $\hat{T}(y)$. We follow the same three steps as above to characterize Pareto-improving tax reforms.

In Step 1, the differential commodity taxes are replaced with a uniform commodity tax. The problem of minimizing expenditure to achieve the status quo level of sub-utility with the reformed commodity taxes for individual h in Step 1 is identical to the previous cases and yields compensated demands $\hat{x}_j((1 + \tau)\mathbf{p}, \omega^h(s))$ for all j as well as the expenditure function (17) and the costs of goods' production (18) hold. That is, it takes fewer resources to supply the bundle of goods needed to attain the status quo sub-utility level for individual h if there are no commodity tax distortions, and this is true for all h.

Step 2 involves reform of the income tax such that all individuals can obtain the status quo utility level. Using the budget constraints (38) and (39), we can write the budget constraint of a person with income y^h as

$$\sum_j q_j x_j = y^h - T(y^h) \tag{40}$$

This person obtains utility $u(v(\mathbf{q}, y^h - T(y^h)), y^h)$ and consumes $\mathbf{x}^h(s)$ in the status quo. After the commodity tax reform, a person who consumes $\hat{x}_j^h$ for all j and earns income y^h will achieve the same utility. The budget constraint that will support this is:

$$\sum_j (1+\tau) p_j \hat{x}_j^h = y^h - \hat{T}(y^h). \tag{41}$$

Therefore, from (40) and (41),

$$\hat{T}(y^h) - T(y^h) = \sum_j \big(1+\tau_j\big) p_j x_j^h(s) - \sum_j (1+\tau) p_j \hat{x}_j^h. \tag{42}$$

The income tax liability changes as the commodity taxes are reformed. If $\tau = 0$, the aggregate income tax liability would have to rise to make up for the loss in aggregate commodity tax revenues. The amount of the income tax change will differ across individuals, reflecting individual's different total consumption expenditure pre- and post-reform. The income tax reform ensures that individuals can achieve their status quo level of sub-utility if they choose their status quo level of income. As individuals' utility depends only on the sub-utility, income and public good, this implies that everyone will choose the same y^h, the new bundle of goods, and be equally well off. Step 3 applies as before. The government has more revenue after the tax reform, and it could use that revenue to obtain a Pareto-improvement.

We have shown that the Atkinson-Stiglitz Theorem – uniform commodity taxation is optimal when preferences over goods are weakly separable – holds when individuals make both extensive and intensive labour supply choices, some individuals may be unemployed and individuals may differ in their preferences for leisure and possibly for the public good. The income tax reform in (42) is, however, very general. To obtain specific results that are comparable to those obtained above, consider the special case where $\phi(\mathbf{x})$ is quasi-homothetic. We obtain the following proposition.

Proposition 4. *Nonlinear Income Tax System (Atkinson and Stiglitz, 1976).*

Assume utility functions for participants and non-participants take the forms $u^i(\phi(\mathbf{x}), y, g)$ *and* $u^0(\phi(\mathbf{x}), 0, g)$ *where sub-utility* $\phi(x)$ *is quasi-homothetic. Let* $(x^h(s), y^h(s), g)$ *be the allocation of individual* h *for* $h = 0, i$ *under an arbitrary nonlinear income tax system* $T(y)$*, with differential commodity taxes* $\boldsymbol{\tau}$*. Another allocation can be implemented using a uniform commodity tax* τ*, a common proportional change in the net-of-tax rate at every income level, and a change in income transfer to those with zero income that gives the same utilities as the allocation* $(\mathbf{x}^h(s), y^h(s), g)$ *and generates more government tax revenues which can be used to make a strict Pareto improvement.*

We again follow the same three steps. In Step 1, the differential commodity taxes are replaced with a uniform commodity tax. The problem of minimizing expenditure to achieve

the status quo level of sub-utility with the reformed commodity taxes for individual h in Step 1 is identical to the previous cases and yields compensated demand $\hat{x}_j((1+\tau)\mathbf{p}, \omega^h(s))$ for all j and the expenditure function (17) and the costs of goods' production (18).

Step 2. *Adjustment of the nonlinear income tax system.*

Given that $\phi(\mathbf{x})$ is quasi-homothetic, we can invert the indirect sub-utility function when there is a uniform commodity tax to obtain

$$e\big((1+\tau)\mathbf{p}, \omega^h(s)\big) = \sum_j (1+\tau)p_j\,\hat{x}_j^h = \frac{\omega^h(s) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)}.$$

Substituting in the expression for $\omega^h(s)$ yields:

$$\sum_j (1+\tau)p_j\,\hat{x}_j^h = \frac{\mu(\mathbf{q}) + \psi(\mathbf{q})\big(y^h - T(y^h)\big) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)}. \tag{43}$$

After the tax reform, an individual who consumes $\hat{x}_j^h$ for all j and earns income $y^h(s)$ will achieve the same utility as in the status quo. The budget constraint that supports this is:

$$\sum_j (1+\tau)p_j\,\hat{x}_j^h = y^h - \hat{T}(y^h) \tag{44}$$

where $\hat{T}(y)$ is the post-reform income tax system. It follows from (43) and (44) that

$$y^h - \hat{T}(y^h) = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)}\big(y^h - T(y^h)\big). \tag{45}$$

By construction, (45) holds for any income level.

To determine how the income tax schedule must be adjusted, we first evaluate (45) at $y^0 = 0$ to obtain

$$-\hat{T}(0) = \frac{\mu(\mathbf{q}) - \mu\big((1+\tau)\mathbf{p}\big)}{\psi\big((1+\tau)\mathbf{p}\big)} + \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)}\big(-T(0)\big). \tag{46}$$

Second, we totally differentiate (45) with respect to income and evaluate the resulting expression at $y = y^h$ to obtain:

$$1 - \hat{T}'(y^h) = \frac{\psi(\mathbf{q})}{\psi\big((1+\tau)\mathbf{p}\big)}\big(1 - T'(y^h)\big) \tag{47}$$

Note that (46) and (47) are analogous to (32) and (33) and have the same interpretation. The net-of-tax rate $1 - \hat{T}'$ is changed in the same proportion at all income levels, and the transfer to non-participants $T(0)$ is adjusted in the same manner as the lump-sum transfer a to change the intercept of the budget set.[12]

Eqs. (46) and (47) show how the status quo income tax system can be reformed with the elimination of differential commodity taxes such that individuals can choose the same income as in the status quo, the vector of commodities that minimize production costs and obtain the same level of sub-utility of goods as in their status quo. As individual utility depends only on the value of this sub-utility, income and public good, individual h who faces the reformed income tax system and a uniform commodity tax will make the same income choices as the status quo. As the commodity choices of individuals uses less resources than the status quo, the government has additional resources that it can use to increase the public good and make everyone better off.

The above result implies the Atkinson-Stiglitz Theorem. If in the initial situation, nonlinear income taxes are optimized but commodity taxes are differential, a Pareto-improving tax reform involving a move to uniform commodity taxes is possible. Therefore, in the full optimum, commodity taxes should be uniform. The level of the optimal uniform commodity tax rate is indeterminate, which implies that the structure of the optimal nonlinear income tax will also be indeterminate. However, the combined commodity tax and income tax schedule will be unique. If the optimal tax structure is in place, the optimum will be preserved if the commodity tax rate is increased and all income tax rates are reduced in the same proportion. As noted by Edwards *et al*. (1994), what is relevant is the effective marginal tax rate on labour income which includes both the income and commodity tax rates.

7. Extensions

7.1. Quasi-homothetic in a subset of goods

We have shown that if preferences over goods are quasi-homothetic, a Pareto improvement can be achieved by replacing differential commodity taxes with a uniform rate and reforming the income tax by adjusting the net-of-tax rate at each income level in the same proportion and the tax liability of those earning zero income. This holds regardless of the assumed structure of the income tax system. We now show it also holds if preferences are quasi-homothetic and identical over only a subset of goods, while preferences for goods outside of the subset may differ among individuals even for those with the same income. In this case, a Pareto improvement can be achieved by eliminating commodity distortions for that subset of goods only, adjusting the net-of-tax rate at each income level proportionately and the tax liability of those with zero earned income (as before), and adjusting the commodity taxes on the remainder of the goods. As the result holds regardless of the assumed structure of the income tax system, we focus on an arbitrary, possibly nonlinear, income tax system.

Suppose utility is weakly separable and quasi-homothetic in a subset of goods $k \in Q$, so can be written as $u^i(\phi(\mathbf{x}_Q), \mathbf{x}_M, y^i, g)$ for participants $i = 1, \cdots, N$ and $u^0(\phi(\mathbf{x}_Q), \mathbf{x}_M, 0, g)$ for nonparticipants, where $\mathbf{x}_Q$ is the vector of goods in the subset Q, $\mathbf{x}_M$ is the subset of the rest of the goods $j \in M$, and $\phi(\mathbf{x}_Q)$ is quasi-homothetic. The allocation obtained under an arbitrary

income tax $T(y)$ and differential commodity taxes can be Pareto-dominated by an allocation where commodity tax rates on goods $\mathbf{x}_Q$ are uniform and the income tax and the commodity taxes on $\mathbf{x}_M$ are suitably reformed.

We show this in Appendix A by following the same three steps as above. In Step 1, we replace the differential commodity taxes on goods in Q with a uniform rate τ and determine the minimum expenditure needed to achieve the status quo level of sub-utility over the subset of Q goods given that individuals choose the status quo level of income and goods outside the subset Q. In Step 2, we adjust the income tax system and commodity taxes on other goods such that for a person who consumes the original x_j^h for all $j \in M$, and who earns income $y^h(s)$ will achieve the same utility as the status quo. Assuming $\phi(\mathbf{x}_Q)$ is quasi-homothetic, the adjustment to the income tax system is again given by (46) and (47), and the adjustment in the commodity prices of the goods outside subset Q is given by

$$\hat{q}_j = \frac{\psi(\boldsymbol{q}_Q)}{\psi\left((1+\tau)\mathbf{p}_Q\right)} q_j, \; j \in M. \tag{48}$$

Under this reformed tax system individuals can choose the same income and commodities outside subset Q as in the status quo as well as the vector of commodities in subset Q that achieves the status quo level of sub-utility $\phi(\mathbf{x}_Q)$ at minimized production costs. Each individual h who faces the reformed tax system and a uniform commodity tax for goods in subset Q will therefore choose the same income and commodity choices for those goods outside of subset Q as the status quo. As individual commodity choices of goods in subset Q use less resources than the status quo, the government has additional resources that it can use to increase the public good and make everyone better off.

A direct implication is that in either the linear progressive or piecewise linear income tax cases, if preferences are quasi-homothetic over a subset of goods, then taxing these goods at a uniform rate Pareto-dominates taxing these goods at differential rates provided commodity prices of all other goods can be proportionally changed and the income tax system reformed as in the case when preferences are quasi-homothetic over all goods. This result holds even if preferences for goods outside of the subset Q differ conditional on income.

If preferences for goods outside of Q are the same conditional on income, then with a fully nonlinear income tax system, no adjustment to the commodity prices outside of the subset Q is needed for this result. To see this, note with weakly separable preferences in a subset of goods Q, (42) can be written as:

$$\hat{T}(y^h) - T(y^h) = \sum_Q (1+\tau_k) p_k \, x_k - \sum_Q (1+\tau) p_k \hat{x}_k^h + \sum_M q_j \, x_j^h - \sum_M \hat{q}_j \, x_j^h.$$

If there is no adjustment to commodity taxes outside of the subset, so $q_j = \hat{q}_j$ for all $j \in M$, then the last two terms on the right-hand side cancel out. The adjustment to the income tax system at any given income is all that is needed to achieve a Pareto improvement and the adjustment given that $\phi(\mathbf{x}_M)$ is quasi-homothetic is as before.[13]

7.2. Proportional reduction in differential commodity taxation

So far, we have considered replacing differential commodity taxation with a uniform rate and showed how the income tax system could be adjusted to achieve a Pareto improvement when preferences are weakly separable in goods versus labour and quasi-homothetic in goods. Kaplow (2006) shows, assuming weak separability of goods versus leisure and a non-linear income tax, that Pareto improvements could be achieved with a proportional reduction in the differential commodity taxes on all goods rather than a full movement to uniform commodity taxation. The logic is that such a commodity tax reform along with an adjustment to the income tax system ensures that individuals receive the same sub-utility over goods as in the status quo, but that the chosen commodity bundle uses less resources to produce than the status quo commodity bundle.

In Appendix B, we demonstrate this result by adjusting the commodity taxes on all goods j by $\alpha \in (0,1)$. Step 1 of this proof is identical to the above and the adjustment needed to the income tax system to achieve a Pareto improvement when preferences over goods are quasi-homothetic is defined as above. Therefore, all of our results generalize to a proportional reduction in differential commodity taxes rather than a full movement to uniform commodity taxation.

8. Concluding remarks

The issue of when commodity tax rates should be uniform has been a recurrent theme in the optimal taxation literature and is of considerable importance for tax policy. Most countries have adopted VAT systems whose tax administration is considerably simplified to the extent that tax rates are uniform (Wilson, 1989; Slemrod, 1990; Belan *et al*., 2008; Gauthier, 2013). The Mirrlees Review (2011) advocated for a fully uniform VAT, relying on the optimal tax literature for justification. In this paper, we provided a synthesis of the case for uniform commodity taxation based on a tax reform approach. While such an approach goes back to the well-known Corlett and Hague (1953) analysis for a representative agent economy, recent papers by Laroque (2005), Kaplow (2006) and Hellwig (2009) adopt a tax reform approach to generalize the Atkinson-Stiglitz and Deaton Theorems on uniform commodity taxation in an optimal income tax setting. They derive circumstances under which, starting with differential commodity taxes and an arbitrary income tax system, a change to uniform commodity taxes and a reform of the income tax system can be Pareto-improving. In the case where the income tax is nonlinear, weak separability of goods versus leisure is sufficient for a Pareto-improving tax reform. If the income tax is linear progressive, a sufficient condition is that preferences be weakly separable in goods versus leisure and quasi-homothetic in goods.

We have synthesized and generalized these approaches in a number of directions. First, we showed how the Pareto-improving tax reform approach to uniform commodity taxes applies to the representative agent case where income taxation is proportional and to the heterogeneous individual case when the income tax is restricted to be piecewise-linear. In the former case, a sufficient condition is that preferences be homothetic in goods, while in the latter case, preferences can be quasi-homothetic in goods.

Second, we showed how a common methodology applies to the tax reforms in all of these cases. This methodology allows us to derive explicit expressions for how the income tax should be reformed when commodity taxes are changed to be uniform.

Third, our analysis shows that the main restriction required to ensure the possibility of a Pareto-improving tax reform to uniform commodity taxes is identical preferences over goods by all individuals. Otherwise, the features of the economy can vary in a number of key ways. Our results apply regardless of the nature of labour supply. Individuals can vary their labour supply along the extensive as well as the intensive margin. Some of them may be involuntarily unemployed. As well, preferences for leisure can vary among individuals unlike in the Atkinson-Stiglitz (1976) and Deaton (1979) analyses. Finally, individuals may choose corner solution in labour supply, for example, when the income tax is piecewise linear.

Fourth, we showed that the results generalize from fully uniform commodity taxation in two ways. First, as Deaton (1979) and Laroque (2005) show for their particular settings, if preferences are weakly separable in only a subset of goods, a reform to commodity tax uniformity over that subset of goods can be Pareto-improving. We also point out that individuals with the same income may also have different preferences over goods outside of the subset. Second, as Kaplow (2006) showed, Pareto-improvements are possible if all commodity taxes are reduced proportionately rather than moving all the way to uniformity. These results apply to the general cases we have considered.

Finally, although all our results are derived starting from arbitrary initial income taxes, they also naturally apply if income taxes are optimal to begin with. The implication is that provided the sufficiency conditions with respect to preferences for goods are satisfied, commodity taxes should be uniform if income taxes are optimal. In other words, a corollary of our tax reform analysis is that the Atkinson-Stiglitz and Deaton Theorems both apply. As well, the Deaton Theorem generalizes to the case of piecewise linear income taxes as we have shown in Boadway and Cuff (2022).

Appendix

A. Weak separability in a subset of goods

Assume there is an arbitrary income tax system, $T(y^h)$ which could be a linear progressive tax system, a piecewise linear tax system or a fully nonlinear income tax system. In the status quo, the sub-utility maximization problem for individual with income y^h and bundle $(\mathbf{x}_M^h, g)$ is:

$$\max_{\mathbf{x}_Q} \phi(\mathbf{x}_Q) \quad \text{s.t.} \quad \sum_Q q_k x_k = y^h - T(y^h) - \sum_M q_j x_j^h$$

The solution gives demands $\mathbf{x}_Q(\mathbf{q}_Q, y^h - T(y^h) - \mathbf{q}_M\mathbf{x}_M^h) \equiv \mathbf{x}_Q^h(s)$ and indirect sub-utility

$$v(\mathbf{q}_Q, y^h - T(y^h) - \mathbf{q}_M\mathbf{x}_M^h) \equiv \omega^h(s). \tag{49}$$

Step 1. *Replacement of differential commodity taxes for the subset of goods.*

Replace the differential commodity taxes on goods in subset Q with a uniform rate τ and consider the minimum expenditure problem for achieving the status quo level of sub-utility over subset Q.

$$\min_{\mathbf{x}_Q} \sum_Q (1+\tau) p_k x_k \quad \text{s.t.} \quad \phi(\mathbf{x}_Q) = \omega^h(s)$$

The solution is $\hat{x}_k(\mathbf{p}_Q, \omega^h(s))$ for all goods $k \in Q$, and total expenditures satisfy

$$e\left((1+\tau)\mathbf{p}_Q, \omega^h(s)\right) = \sum_Q (1+\tau) p_k \hat{x}_k\left(\mathbf{p}_Q, \omega^h(s)\right) < \sum_Q (1+\tau) p_k x_k(s). \tag{50}$$

The level of sub-utility, $\phi(\hat{\mathbf{x}}_Q^h(\mathbf{p}_Q, \omega^h(s))$, is the same as in the status quo.

Step 2. *Adjustment of income tax system and commodity taxes on other goods.*

Since ϕ is quasi-homothetic, we can invert the indirect sub-utility function for the subset of goods in Q when there is a uniform commodity tax to obtain

$$e\left((1+\tau)\mathbf{p}_Q, \omega^h(s)\right) = \sum_Q (1+\tau) p_k \hat{x}_k^h = \frac{\omega^h(s) - \mu\left((1+\tau)\mathbf{p}_Q\right)}{\psi\left((1+\tau)\mathbf{p}_Q\right)}.$$

Substituting in (49) for $\omega^h(s)$ yields:

$$\sum_Q (1+\tau) p_k \hat{x}_k^h = \frac{\mu(\mathbf{q}_Q) + \psi(\mathbf{q}_Q)(y^h - T(y^h) - \mathbf{q}_M\mathbf{x}_M^h) - \mu\left((1+\tau)\mathbf{p}_Q\right)}{\psi\left((1+\tau)\mathbf{p}_Q\right)} \tag{51}$$

After the tax reform, a person who consumers $\hat{x}_k$ for all $k \in Q$ and the original x_j^h for all $j \in M$, and who earns income $y^h(s)$ will achieve the same utility. The budget constraint that will support this is:

$$\sum_{Q}(1+\tau)p_k\,\hat{x}_k^h = y^h - \hat{T}(y^h) - \sum_{M}\hat{q}_j\,x_j^h. \tag{52}$$

Eqs. (51) and (52) imply that

$$y^h - \hat{T}(y^h) - \hat{\mathbf{q}}_M\mathbf{x}_M^h =$$

$$\frac{\mu(\mathbf{q}_Q) - \mu\left((1+\tau)\mathbf{p}_Q\right)}{\psi\left((1+\tau)\mathbf{p}_Q\right)} + \frac{\psi(\mathbf{q}_Q)}{\psi\left((1+\tau)\mathbf{p}_Q\right)}(y^h - T(y^h) - \mathbf{q}_M\mathbf{x}_M^h) \tag{53}$$

Consider a proportional adjustment to the consumer prices of the goods outside of the subset Q,

$$\hat{q}_j = \frac{\psi(\mathbf{q}_Q)}{\psi\left((1+\tau)\mathbf{p}_Q\right)}q_j,\ j \in M. \tag{54}$$

With this adjustment, the last term on both sides of (53) cancel and the resulting expression is equivalent to (45). This implies that a proportional change in the commodity prices of the goods outside subset Q and an income tax reform given by (46) and (47) will ensure that individuals can choose the same income and vector of commodities outside subset Q as in the status quo and choose the vector of commodities in subset Q that minimize production costs and obtain the same level of sub-utility over goods in subset Q as in the status quo.

B. Pareto Improving Commodity Tax Reductions

Following Kaplow (2005), consider a reform to the differential commodity taxes $\hat{\tau}_j = \alpha\tau_j$ with $\alpha \in (0,1)$ for all j. Step 1 becomes

$$\min_{\{x_j\}}\sum_j(1+\alpha\tau_j)p_j\,x_j \quad \text{s.t. } \phi(x_1,\cdots,x_m) = \omega^h(s)$$

which yields $\hat{\mathbf{x}}((1+\alpha\tau_j)p_j, \omega^h(s))$. The income tax is reformed so that with the same income the individual can just afford this commodity bundle, that is

$$y^h - \hat{T}(y^h) = \sum_j(1+\alpha\tau_j)p_j\,\hat{x}_j^h. \tag{55}$$

By revealed preference, it must be

$$y^h - \hat{T}(y^h) =$$

$$\sum_j(1+\alpha\tau_j)p_j\,\hat{x}_j^h < \sum_j(1+\alpha\tau_j)p_j\,x_j^h(s) = \sum_j p_j\,x_j^h(s) + \sum_j \alpha\tau_j\,p_j x_j^h(s) \tag{56}$$

where $x_j^h(s)$ is the status quo commodity choice for all j. The status quo budget constraint is

$$y^h - T(y^h) = \sum_j (1+\tau_j) p_j \, x_j^h(s) \tag{57}$$

Solving (57) for $\sum_j p_j x_j^h(s)$, substituting into (56) and re-arranging yields

$$\frac{1}{1-\alpha}\left(\hat{T}(y^h) - T(y^h)\right) > \sum_j \tau_j \, p_j x_j^h(s) \tag{58}$$

Next, recall that $\hat{\mathbf{x}}^h$ and $\mathbf{x}^h(s)$ yield the same sub-utility. Therefore, at the original prices $\mathbf{x}^h(s)$ was chosen, which implies (by revealed preference) $\hat{\mathbf{x}}^h$ was not affordable, that is,

$$\sum_j (1+\tau_j) p_j \, \hat{x}_j^h > \sum_j (1+\tau_j) p_j \, x_j^h(s). \tag{59}$$

Solving both (55) and (57) for y^h and setting equal yields:

$$T(y^h) + \sum_j (1+\tau_j) p_j \, x_j^h(s) = \hat{T}(y^h) + \sum_j (1+\alpha\tau_j) p_j \, \hat{x}_j^h \tag{60}$$

Using (60) to substitute out for $\sum_j p_j \, \hat{x}_j^h$ in (59) and simplifying yields

$$\sum_j \alpha\tau_j \, p_j \hat{x}_j^h > \frac{\alpha}{1-\alpha}\left(\hat{T}(y^h) - T(y^h)\right). \tag{61}$$

Adding the right-hand side terms and the left-hand side terms of (61) and (58) we obtain

$$\hat{T}(y^h) + \sum_j \alpha\tau_j \, p_j \hat{x}_j^h > T(y^h) + \sum_j \tau_j \, p_j x_j^h(s) \tag{62}$$

For every y^h, the reformed income tax system ensures individuals have the same level of sub-utility and income as the status quo and raises more tax revenue. Summing (61) up over all individuals yields the government's budget constraints in the reformed and status quo allocations which using the individual budget constraints can be rewritten as

$$\sum_h \sum_j p_j \hat{x}_j^h < \sum_h \sum_j p_j \, x_j^h(s).$$

Fewer resources are needed in the reformed allocation than in the status quo.

Notes

1. Exceptions include Christiansen (1984), Edwards *et al*. (1994), Nava *et al*. (1996) and Jacobs and Boadway (2014), all of whom derive conditions analogous to the Corlett-Hague Theorem for characterizing the direction in which commodity taxes should differ from uniformity.

2. Deviations from these assumptions would lead to a violation of the Atkinson-Stiglitz Theorem, so differential commodity tax should be used alongside the optimal income tax. Examples of such violations include differences in preferences for goods (Saez, 2002), differences in needs (Boadway and Pestieau, 2003), differences in endowments (Cremer *et al*., 2001), endogenous wages (Naito, 1999) and externalities (Pirttilä and Tuomala, 1997). In fact, differential commodity taxation that is used in practice typically serves some externality corrective purposes, e.g., excise taxes on gasoline, tobacco, alcohol, and recently sugar-sweetened beverages. The weak separability assumption has also been challenged empirically. See, for example, Browning and Meghir (1991).

3. Crawford *et al*. (2011, p 350) note that 'the available empirical evidence firmly rejects weak separability. [However,] the departures from weak separability, while significant, are quantitatively small [which] suggests that the gains from differentiation may not be large [...]. There is some evidence, and much anecdotal wisdom, that implementing such rate differentiation through the VAT would be quite costly'.

4. Christiansen (1984) adopted a similar tax reform approach to examine the conditions under which deviations from uniform commodity taxation combined with a revenue-neutral adjustment to the optimal income tax system would be desirable. He demonstrates that weak separability implies consumption does not depend on leisure for a given amount of income. He shows that if the consumption for some good increases (decreases) with leisure holding income constant, then the good should be taxed (subsidized).

5. We follow the convention of using superscripts to indicate individual types, including the income tax bracket of the individual's income, and subscripts to refer to goods and partial derivatives.

6. Specifying utility in terms of income y instead of labour or leisure is for simplicity only. Income is proportional to labour by the given wage rate, and leisure is total hours available less hour worked.

7. We could distinguish the utility of non-participants by individual, given that non-participants can include individuals with different productivities or preferences. Since that would simply complicate notation without affecting results, we let the utility of all non-participants be identical and given by $u^0(\cdot)$.

8. Sandmo (1976) assumed the utility function was homothetic in all goods and leisure. However, as our tax reform analysis indicates, homotheticity in goods alone is sufficient to ensure commodity taxes are uniform in the optimum.

9. Using (13) with $\tau = 0$ and the post-reform budget constraint (14), the uniform commodity rate needed for the tax reform in Step 2 is given by $1+\hat{\tau}=\psi(\mathbf{p})/\psi(\mathbf{q})$.

10. Formally, the government budget constraint combined with individual budget constraints yields the economy's aggregate resource constraint.

11. The needed uniform commodity tax rate is the same as in the case with identical individuals, that is, $1+\hat{\tau}=\psi(\mathbf{p})/\psi(\mathbf{q})$.

12. Another way to see this is to linearize the pre- and post-reform budget constraints to obtain: $(1-T'(y))y+E$ and $(1-\widehat{T}'(y))y+\widehat{E}$ where E and $\widehat{E}$ are the 'virtual incomes' in the pre- and post-reform tax system, respectively. Substituting these linearized budget constraints and (47) into (45) yields (46) with $-T(0)$ and $-\widehat{T}'(0)$ replaced by E and $\widehat{E}$, respectively.

13. This result holds regardless of form of $\phi(\mathbf{x}_M)$ (see Remark 3 of Laroque, 2005). Laroque (2005) considered the case of utility being weakly separable in two sub-utility functions defined over two different sets of goods. Each of the sub-utility functions are the same for all individuals. Having commodity prices proportional to producer prices within each subset generates a Pareto improvement under a suitable adjustment to the nonlinear income tax system.

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Resumen

Generalizando el enfoque de reformas fiscales de Laroque (2005), Kaplow (2006) y Hellwig (2009), mostramos que para cualquier sistema de imposición sobre la renta, la imposición diferencial sobre las mercancías puede ser dominada en el sentido de Pareto por una reforma que grave todas las mercancías a un tipo único, siempre y cuando se convine con un impuesto sobre la renta adecuado. A partir de un modelo de agente representativo, bajo determinadas especificaciones particulares de preferencias, derivamos las reformas necesarias en un impuesto sobre la renta lineal progresivo, lineal por tramos y no lineal. Las reglas de imposición óptima de un impuesto sobre el consumo uniforme de Sandmo (1974), Atkinson y Stiglitz (1976) y Deaton (1979) se mantienen, en presencia de desempleo involuntario y cuando los individuos tienen diferentes preferencias por el ocio y aumentan la participación extensiva en el mercado de trabajo.

Palabras clave: imposición uniforme sobre el consumo, impuesto óptimo sobre la renta, impuesto sobre la renta lineal por tramos.

Clasificación JEL: H21, H23, H24.