



# Innovation and Competition in a Mixed Oligopoly\*

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## Abstract

We consider a mixed quantity-setting oligopoly where firms can innovate to reduce their marginal cost of production. Besides, private firms may also reduce output competition through a collusive agreement. In this context, we obtain that collusion incentives are weaker due to R&D activities. We also investigate two different regulatory measures; (possibly partial) privatization and an output subsidy. In the latter case, we obtain that when firms innovate, the privatization neutrality result is not satisfied. Furthermore, a proper policy should include a partial privation where less competition between private firms calls for a weaker privatization scheme.

**Keywords:** Imperfect competition, mixed oligopoly, partial privatization, innovation, subsidies.

**JEL Classification:** L11, L13, L52, D43.

## 1. Introduction

The literature analyzing the social benefits and costs associated with the existence of public or at least to some extent state-owned firms is extensive. These type of firms are indeed very common in many markets such as airlines, telecommunications, railways, electricity, banking, broadcasting, and education. Even though many papers have examined mixed oligopolies considering, for instance, whether privatization or subsidization is optimal, the

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results are often mixed. Concerning the effects of subsidization, it has been shown that there are no welfare consequences from the privatization of a public firm whenever a subsidy ensures the first-best allocation. In other words, welfare is the same before and after privatization when the government optimally subsidizes firms. For instance, White (1996) shows that in a Cournot setting both in the private and in the mixed oligopoly, the optimal subsidy is identical. This has often been referred to as the privatization neutrality theorem. Some other papers, though, have derived a non-neutral result of privatization. For example, assuming Cournot competition and no regulation apart from the operation of the public firm, De Fraja and Delbono (1989), (see also, De Fraja and Delbono, 1990 for a review of this literature) obtain that privatization will lower welfare if there are relatively few private firms in the market and will raise welfare if there are relatively many private firms. The intuition is that privatization might increase welfare since with a large number of private firms the public firm must produce a very high level of output, driving private profits to a very low level. In the same line of research, Matsumura and Tomaru (2012, 2013) have respectively shown that privatization neutrality does not hold if there are foreign competitors or when an excess burden of taxation is introduced. Additionally, in a recent paper, Lin and Matsumura (2018) have shown that the neutrality result does not hold unless public and private firms have the same cost function.

To the best of our knowledge, previous studies on policy issues in mixed oligopolies have mostly ignored the competitive (or collusive) behavior of private firms and that public firms are often important players in R&D intensive industries such as health-care, energy or bio-agriculture (see Kesavayuth and Zikos, 2013). Among the few exceptions are the papers by Matsumura and Okamura (2015), Escrihuela-Villar and Gutiérrez-Hita (2018a, 2018b) and Gil-Moltó *et al.* (2011) with which the present work is most closely related. In the first work, the authors introduce an interdependent payoff structure into a mixed oligopoly assuming that firms consider their own and other firms' profits. It is obtained that the optimal degree of privatization is higher when there is less market competition. In the same line, Escrihuela-Villar and Gutiérrez-Hita (2018a, 2018b), study the optimal government intervention contrasting different regulatory timings and a bias toward consumer surplus in the public firm maximization problem respectively. Their main conclusion is that the privatization neutrality result is not robust to the existence of cooperation between the private firms. On the other hand, the work by Gil-Moltó *et al.* (2011) mainly obtains that privatization reduces R&D activity and welfare. There are, however, some important differences between the present paper and the above-mentioned papers. To begin with, Matsumura and Okamura and Escrihuela-Villar and Gutiérrez-Hita do not consider innovation. Regarding Gil-Moltó and coauthors, they mainly restrict attention to Cournot competition ignoring, therefore, the effects of (or on) the intensity of market competition. They also do not consider partial privatization. We believe that these considerations are important for several reasons. For instance, providing some empirical examples, Lee *et al.* (2018) argue that partial privatization through mixed public-private firms is increasingly used in several European countries and that regulators use mixed firms following cost considerations or financial constraints<sup>1</sup>. On the other hand, the effect of private firms' collusion in mixed oligopolies is also important. Some empirical studies have considered how weak competition makes it difficult for local governments to obtain benefits

from contracting out. For instance, in the Netherlands, Dijkgraaf and Gradus (2007) investigate whether collusion exists and what the impact is on tariffs for waste collection. They show the existence of collusion between private firms and, as a consequence, the presence of several competing public firms might be essential to ensure more and fair competition. The electricity market provides also several interesting examples; in the 90s United States, Chile, and other countries within EU (Great Britain, Spain, Germany, among others) implemented reforms aimed to privatize and restructure the electric power industry. Several authors have found that wholesale prices increased and that some degree of collusion could be observed in these markets (see, for instance, Wolfram, 1999 or Borenstein and Bushnell, 1999).

Summarizing, the main goal of the present paper is precisely to study both the optimal privatization as well as the output subsidization in a mixed oligopoly where firms can also strategically innovate. To that extent, we firstly develop as a benchmark model a mixed oligopoly where private firms simultaneously decide the amount of R&D undertaken. Later, and following (among many others) Matsumura and Okamura (2015), we assume that firms compete in quantities with the particularity of an interdependent payoff structure in such a way that private firms maximize the sum of their profits and a fraction of the other private firm's profits<sup>2</sup>. Consequently, this fraction may be considered as the degree of competition which implies that firms can agree on a distribution of the output quotas different to that arising from a perfect joint profit maximization agreement<sup>3</sup>. In other words, this way of modeling the intensity of competition, that has received growing attention of scholars<sup>4</sup>, consists of assuming that each firm cares about its profit plus a weighted average of the profits of the other firms. This formulation is closely related to the *coefficient of cooperation*, defined by Cyert and DeGroot (1973) and it is also in line with the growing and more recent behavioral economics literature, as well as with experimental games that test that subjects are concerned with reciprocity (see for instance Fehr and Schmidt, 1999 and Charness and Rabin, 2002 respectively). The notion of collusion employed here also resembles the model presented in Reynolds and Snapp (1986) and Farrell and Shapiro (1990) who considered the competitive effects of partial ownership of rival firms. In the context of a single-period Cournot oligopoly model, they show that as the degree of cross-ownership among rivals increases the equilibrium in the market becomes less competitive in the sense that aggregate output falls toward the monopoly level. In the present formulation, the idea of cross-ownership is replaced by an explicit reciprocal concern that leads firms to a collusive outcome. Additionally, Joh (1999) presents some empirical results supporting our approach. In her work, using data from Japanese firms during the period from 1968 to 1992, she shows that when compensation is positively linked to the industry performance through, for instance, a strategic group performance evaluation, the sustainability of a collusive outcome increases.

Our first result corroborates the results of Escrihuela-Villar and Gutiérrez-Hita (2018a, 2018b) that collusion profitability is limited by the output expansion of the public firm intended to maximize welfare. We add to the literature obtaining that collusion incentives are weaker due to R&D activities. This result confirms the findings of several empirical studies. For instance, Gerosky and Pomroy (1990) show that innovating may be a way to obtain market power. In particular, he finds that innovation increases the degree of competition which

leads to a fall in market concentration over time. In the same line, using a panel of Italian industrial sectors for the period 1978-1993, Paci and Usai (1998) confirm previous evidence from other countries where technological efforts prove to have a negative impact on the level of concentration. In a simulation study, Meyer *et al.* (1996) state that one can speak of an intensification of competition due to R&D activities, which is in contrast to the traditional interpretation of the Schumpeterian relation between concentration and R&D. Secondly, we also obtain that even though full privatization would result in a reduction in total welfare, there is always an optimal degree of partial privatization. This result, though, is subject to the definition of the objective function of the public firm since a larger pro-consumer bias would call for a lower privatization scheme. Finally, we also obtain that the neutrality result is not satisfied when firms innovate regardless of the degree of competition between private firms. In other words, we establish that the main results in Lin and Matsumura (2018) or Escrihuela-Villar and Gutiérrez-Hita (2018b) critically hinge on the assumption that firms do not innovate. Therefore, under the realistic assumption that firms in a mixed oligopoly innovate, we identify plausible conditions under which an optimal privatization scheme is important despite the presence of optimal production subsidies.

The rest of the paper is structured as follows. Section 2 describes the basic model where firms innovate in an imperfectly collusive mixed market, and it considers the welfare effects of partial privatization. Section 3 considers an optimal production subsidy to establish the validity of the privatization neutrality result. Section 4 concludes. The appendix includes an outline of the proofs. We included the development of the model and the full proofs in an online appendix.

## 2. The benchmark model: Collusion in a mixed oligopoly with R&D

We consider an industry with three firms indexed by  $i=1,2,3$  simultaneously producing a homogeneous product. Two of these firms indexed by  $i=2,3$  are profit-maximizing private firms whereas the firm indexed by 1 is assumed to be a welfare-maximizing public firm. Welfare ( $W$ ) accounts for cumulative firm's profits  $\sum_{i=1}^3 \Pi_i$  plus consumer surplus ( $CS$ ), where  $\Pi_i$  denotes profit of firm  $i$ . Industry inverse demand is piecewise linear  $p(Q)=\max(0, a-Q)$ , where  $Q = \sum_{i=1}^3 q_i$  is the industry output,  $p$  is the output price, and  $a>0$ . We assume that entry and exit in the market are not possible. In many cases, though, a partial privatization structure can be observed and, therefore, the government still holds a positive proportion of shares in privatized firms. Therefore, these semi-privatized firms cannot be pure welfare maximizers. In the present framework, we assume that the unique semi-public firm maximizes the weighted sum of own profit and welfare:

$$\beta(\sum_{i=1}^3 \Pi_i + CS) + (1 - \beta)\Pi_1 \quad (1)$$

where  $\beta \in [0,1]$ . Consequently, the present formulation encompasses the model with one pure public firm and two private firms if  $\beta=1$  and the quantity competition among three private firms if  $\beta=0$ .

The present model follows Gil-Moltó *et al.* (2011) where, to avoid situations in which private firms are driven out of the market, the existence of diminishing returns to scale is assumed by introducing a quadratic term related to production in the firms' cost function. Then, firm  $i$ 's total cost function depends on its level of production, and on the level of R&D, that we denote by  $x_i$ ; then,  $c(q_i, x_i) = (c - x_i)q_i + q_i^2$ .<sup>5</sup> As stated in De Fraja and Delbono (1990), if each firm's marginal cost is constant, the public firm will impose the rule of pricing at marginal cost. Then, if there were any fixed costs, the public firm would be unable to cover the losses which would then need to be funded by the taxpayer. This way of modeling a cost function also reflects the fact that the public firm is "ex-ante" equally as efficient as the private firms. In other words, without R&D and for a given quantity, the cost of production is the same for all firms. R&D has also a cost given by  $\zeta(x_i) = \gamma x_i^2$  which reflects the diminishing returns to R&D investment. Firm  $i$  profit function can be written like

$$\Pi_i = (a - Q)q_i - (c - x_i)q_i - q_i^2 - \gamma x_i^2, i = 1, 2, 3 \text{ and } j \neq i. \quad (2)$$

We assume in our model that R&D is strategic<sup>6</sup>: In the first stage, all firms simultaneously and non-cooperatively choose R&D levels where the private firms maximize individual profits with respect its R&D level  $x_i$  whereas the semi-public firm also considers (to some extent) total welfare. These R&D levels are made known to all firms, and then in the second stage, output levels are also simultaneously determined in the market. As mentioned in the introduction, we characterize imperfect collusion in the output stage considering a particular model where, in the second stage, private firms maximize the sum of own profits and a fraction of the profits of the other private firm. Explicitly, in the second stage firm  $i=2,3$  maximizes  $\Pi_i + \alpha \Pi_j$  where  $\alpha \in [0,1]$ . The parameter  $\alpha$ , that we assume to be constant and symmetric, can be interpreted as representing the degree of reciprocal preference. A positive  $\alpha$  is equivalent to firms having reciprocal payoff functions and therefore may represent the degree of imperfect collusion in the output market. Equivalently, a direct link between a positive  $\alpha$  and the degree of collusion between the private firms can also be established. Admittedly, since  $\alpha$  is common knowledge, one could interpret that the public firm and the regulator know that private firms are (imperfectly) illegally colluding. An alternative interpretation is that our parameter  $\alpha$  might also be a proxy for a set of characteristics that shape the degree of competition between private firms including (but not limited to) collusion.

We also assume imperfect collusion only between the private firms whereas the (semi) public firm is concerned with social welfare and individual profits. In the present model, then, even with  $\beta=0$ , the former public firm does not cooperate with the private firms. We believe that this assumption is reasonable, at least in the short run, since cooperation often involves an agreement between firms that can easily coordinate with each other because they are of a similar type or have a common corporate culture (see, for instance, Levenstein and Suslow, 2006). The non-cooperating or fringe firms often consists of several foreign firms or new entrants that could not coordinate their behavior with the colluding firms even if they wish so<sup>7</sup>.

**Definition 1.**—*Collusion is said to be imperfect if  $\alpha \in (0,1)$ . On the contrary, collusion is said to be perfect if  $\alpha=1$ .*

Even though private firms might (imperfectly) collude with respect to the output produced, we assume that the level of R&D is determined non-cooperatively. In other words, firms might reach a collusive agreement consisting of an output vector with productions  $q_2(\alpha, x_1, x_2, x_3)$  and  $q_3(\alpha, x_1, x_2, x_3)$  that depend on the R&D previously decided in a non-cooperative way. As usual, the model is solved by backward induction. Hence, the solution of the second stage of the game is obtained when firm  $i$ ,  $i=2,3$  and  $j \neq i$  maximizes  $\Pi_i + \alpha \Pi_j$  with respect to  $q_i$  where  $\Pi_i$  is defined in (2), whereas firm 1 maximizes (1). We also note that since  $2\gamma$  is the slope of the marginal cost of implementing R&D, an increase in  $\gamma$  can be directly linked to a reduction in the innovative activities, and therefore the limit case where  $\gamma \rightarrow \infty$  implies that firms no longer innovate<sup>8</sup>.

This leads us to the following linear downward sloping reaction functions:

$$\begin{aligned} q_1 &= \frac{(a - c - q_2 - q_3 + x_1)}{4 - \beta} \\ q_i &= \frac{(a - c - q_j(1 + \alpha) - q_1 - x_i)}{4} \text{ for } i = 2, 3 \text{ and } j \neq i. \end{aligned} \quad (3)$$

We note that an increase in  $\alpha$  increases the (absolute value) of the slope of the reaction function of private firms. Consequently, for a given symmetric R&D equilibrium, the quantities produced by each private firm will be lower if  $\alpha$  increases. However, the effect of  $\alpha$  on  $q_i$  also depends on the effect of  $\alpha$  on the level of R&D undertaken and determined at the first stage of the game. The intersection of the reaction functions leads to the Nash equilibrium of the second stage for given values of the R&D activities:

$$\begin{aligned} q_1 &= \frac{(a-c)(3+\alpha)+x_1(5+\alpha)-(x_i+x_j)}{18+4\alpha-\beta(5+\alpha)} \\ q_i &= \frac{x_j(15-4\beta)+x_i(\alpha(\beta-4)+\beta-3)+x_1(\alpha-3)}{(\alpha-3)(\alpha(\beta-4)+5\beta-18)} + \frac{(a-c)(\beta-3)}{\alpha(\beta-4)+5\beta-18} \text{ for } i = 2, 3 \text{ and } j \neq i. \end{aligned} \quad (4)$$

To solve the first stage of the game, we have to replace the quantities above in the profit functions described in (2). Then, firms 2 and 3 maximize individual profits with respect to  $x_2$  and  $x_3$  respectively and firm 1 maximizes (1) with respect to  $x_1$  to obtain three reaction functions. The intersection of these reaction functions reports the symmetric non-cooperative optimal level of R&D, the imperfectly collusive production levels, and the associated profits of this two-stage game. It can be easily checked that the public firm's output increases with  $\alpha$  and decreases with  $\beta$  while the reverse is true for private firms.

As stated in Escrihuela-Villar and Gutiérrez-Hita (2018a), the present formulation of imperfect collusion is mathematically equivalent to a model of standard strategic delegation. In this case, since firms produce substitute goods, it is well-known that a firm has a strategic incentive for committing to a larger output than the profit-maximizing level to reduce the rivals' outputs. The aforementioned output expansion by the public firm might also constrain the incentives of private firms to reduce competition in our model. We extend Proposition 1 in

Escrighuela-Villar and Gutiérrez-Hita (2018a) to the case where firms can also perform R&D activities.

**Proposition 1.**—*The symmetric private firms' profit-maximizing degree of imperfect collusion is given by  $\hat{\alpha}$  where  $\hat{\alpha} < 1$ . Besides, if  $\beta = 1$ ,  $\hat{\alpha}$  decreases with  $\beta$  and increases with  $\gamma$  and  $\lim_{\gamma \rightarrow \infty} \hat{\alpha} = \frac{\beta-2}{\beta-4} < 1$ .*

As mentioned above, private firms reduce their output to increase the market price if the degree of imperfect collusion increases<sup>9</sup>. On the contrary, the reaction of the public firm to an increase in  $\alpha$  is an output expansion that potentially limits the scope and, consequently, the profitability of imperfect collusion. Furthermore, when a public firm is partially privatized, the (former) public firm cares less about welfare but more about its profits. Therefore, the output expansion effect of the public firm is also mitigated. These effects have already been captured by Escrighuela-Villar and Gutiérrez-Hita (2018a) in the absence of R&D. Proposition 1 adds to this result by showing that when firms can undertake innovative activities, collusion incentives are also reduced. Interestingly enough, this is true regardless of the degree of privatization of the public firm. The intuition is that the effect of the strategic R&D aimed to lower one's own marginal cost leads to an increase in production that increases the degree of competition in the market. Roughly speaking, more efficient firms have potentially fewer incentives to cut production through a collusive agreement<sup>10</sup>.

Additionally, since  $\lim_{\gamma \rightarrow \infty} \hat{\alpha}$  describes private firms' profit-maximizing degree of imperfect collusion in the absence of R&D activities, and as long as  $\hat{\alpha}$  increases with the R&D cost  $\gamma$ , we can derive the following corollary.

**Corollary 1.**—*The symmetric profit-maximizing degree of imperfect collusion is lower when private firms undertake R&D activities than in the absence of them.*

In other words, collusion incentives are weaker due to R&D activities.

As mentioned in the introduction, the literature has extensively examined mixed oligopolies considering whether privatization is optimal. However, this literature becomes thinner if one moves into the context where firms perform R&D. As an exception, Gil-Moltó *et al.* (2011) find that full privatization of the public firm reduces R&D activity and welfare in the duopoly market. Nevertheless, they consider neither the degree of imperfect collusion between private firms nor the possibility of partial privatization. We consider next the effects of (partial) privatization on total welfare.

**Proposition 2.**—*In our mixed oligopoly with imperfect collusion and innovative firms, one has:*

- (i) *W is larger at  $\beta=1$  than at  $\beta=0$ .*
- (ii) *W is maximized when  $\beta = \hat{\beta} \in (0, 1)$ .*



$$(iii) \left. \frac{\partial \hat{\beta}}{\partial \alpha} \right|_{\alpha=0} < 0. \text{ More specifically, if } \alpha=0, \text{ then } \hat{\beta}=0.695 \text{ and if } \alpha=1, \text{ then } \hat{\beta}=0.655.$$

For the case without spillovers and  $\alpha \geq 0$ , Proposition 2 corroborates Proposition 6 in Gil-Moltó *et al.* (2011) where full privatization results in a reduction in total welfare. The present paper goes one step further and also considers partial privatization to argue that there is a degree of partial privatization that maximizes welfare. The interaction of different forces that go in different directions explains Proposition 2. Firstly, since a decrease in  $\beta$  implies a reduction in the welfare-maximizing output expansion incentives of the semi-public firm, consumers' surplus increase with  $\beta$ . At the same time, this effect enhances private firms' profits, which always decrease with  $\beta$ . Finally, there is an inverse U-shaped relationship between public firm's profits and the degree of nationalization. The intuition is simple. Departing from  $\beta=1$ , an increase in the degree of privatization (implying an output contraction of the large output produced by the public firm) increases public firm's profits through the subsequent increase in the market price. However, if  $\beta$  decreases beyond a certain level and since private firms react to the decrease in  $\beta$  by expanding their output, the semi-public firm does not benefit anymore from the decrease in  $\beta$ . Additionally, we obtain that, in the absence of collusion, an increase in private firms' cooperation leads to a larger degree of optimal partial privatization. The intuition is as follows. When private firms collude, the aforementioned positive effect of a decrease in  $\beta$  on private firms' profits becomes also larger whereas the effect of  $\beta$  on consumers' surplus and public firm's profits become weaker. The first force compensates the second ones.

Admittedly, our last result crucially depends on the objective function of the regulator. The public firm might have a certain political orientation with an objective function different from the standard welfare-maximizing one taking thus a wide variety of forms ranging from pro-consumer to pro-business (see, for instance, White, 2002 or Escrhuella-Villar and Gutiérrez-Hita, 2018b). More precisely, since  $CS$  always increases with  $\beta$  and, furthermore, this effect becomes more significant if private firms increase their degree of collusion (namely,  $\partial^2 CS / \partial \beta \partial \alpha > 0$ ), an increase in the degree of imperfect collusion would call for a weaker privatization scheme due to the bias towards  $CS$  in the public firm maximization problem.

### 3. Output subsidy

In this section, we investigate the extent to which a welfare-maximizing social planner can mitigate the negative effect that the degree of imperfect collusion among private firms might have in welfare. We study the implementation of an optimal production subsidy, that we denote by  $s$ , for all firms where, in this case, welfare must also incorporate the cost of the subsidy. We assume thus that the government maximizes cumulative firm's profits plus  $CS$  but also taking into account that the subsidy is an expenditure that has to be financed. To put it differently, the subsidy is included in total welfare as part of the public and private firms' profits but also as an equivalent expenditure in such a way that the government now maximizes  $W^s \equiv \sum_{i=1}^3 \Pi_i + CS - s(\sum_{i=1}^3 q_i)$ . More precisely, the game at hand can be modeled as



follows: firstly, for given values of  $\alpha$  and  $\beta$ , the social planner decides the level of the subsidy  $s$  that maximizes welfare. Secondly, and as in the previous section, firms simultaneously decide their R&D levels. Finally, in the third stage, the output levels are also simultaneously determined in the market for the values of  $s$  previously determined.

As mentioned in the introduction, the privatization neutrality theorem states that, under some circumstances, there are no consequences from the privatization of a public firm whenever a subsidy ensures the first-best allocation. In this line of research, Escribuela-Villar and Gutiérrez-Hita (2018b) show that the privatization neutrality result is not satisfied whenever there is at least some cooperation between the private firms. We next examine the role of the R&D activities in the privatization neutrality result.

**Proposition 3.**—*In our mixed oligopoly with imperfect collusion, innovative firms and an optimal production subsidy, one has:*

- (i) if  $\alpha \in [0, \tilde{\alpha})$  where  $\tilde{\alpha} \approx 0.307$ ,  $\frac{\partial W^s}{\partial \beta} < 0$
- (ii) if  $\alpha \geq \tilde{\alpha}$ ,  $W^s$  is maximized at  $\beta = \tilde{\beta} \in (0, 1)$  where  $\tilde{\beta}$  increases with  $\alpha$ . Moreover,  $\tilde{\beta} \approx 0.83$  if  $\alpha = 1$ .

Proposition 3 shows that, regardless of the degree of competition between private firms, the neutrality result is not satisfied when firms innovate. Consequently, if firms innovate, privatization affects welfare despite the inclusion of a welfare-maximizing production subsidy. Conversely, and corroborating Proposition 1 in Escribuela-Villar and Gutiérrez-Hita (2018b), it can be easily checked that if  $\alpha = 0$ ,  $\lim_{\gamma \rightarrow +\infty} W^s = \frac{3(a-c)^2}{10}$ . In words, if firms do not innovate, the privatization neutrality result is satisfied when firms compete à la Cournot since, in this case, welfare does not depend on  $\beta$ . Summarizing, our analysis proves useful to identify that the main results in Lin and Matsumura (2018) or Escribuela-Villar and Gutiérrez-Hita (2018b) crucially rely on the assumption that firms do not innovate.

**Remark 1.**—*In our mixed oligopoly with innovative firms and an optimal production subsidy, total welfare always depends on  $\beta$  for all  $\alpha \in [0, 1]$ .*

Proposition 3 analyzes the effect of the degree of privatization on welfare when firms have an optimal production subsidy. We obtain that when collusion is relatively low, privatization increases welfare. We could interpret it as follows; in the absence of a significant distortion provoked by firms' collusion, an output subsidy guarantees optimality in a private oligopoly, and there is no need for the government's participation in the firm. Conversely, if collusion between private firms is relatively large, full privatization does not maximize welfare, and the government should keep a certain number of shares of the public firm. Additionally, and even though full nationalization does not maximize welfare even in the limit case of perfect collusion, less competition between private firms always calls for a weaker privatization scheme which contrasts with Proposition 2 where production subsidies were not included. Interestingly enough, Proposition 3 shows that when we introduce an optimal

subsidy, and since optimality might (generally) not be obtained without a proper privatization scheme, the existence of a public firm is not recommended whenever private firms sufficiently compete with each other. However, when the degree of collusion between private firms is large enough, one might interpret that a production subsidy should be complemented with a regulation ensuring large participation of the public sector in the market.

#### 4. Concluding comments

We developed a theoretical framework to study a mixed oligopoly market in which private firms might collude in the output produced. We have modeled imperfect collusion between private firms using an interdependent payoff structure where private firms also care, to some extent, about the profits of a rival firm. Contrary to previous studies, we consider in our framework also innovative firms. In this sense, an exception is Gil-Moltó *et al.* (2011), where they analyze full privatization and R&D in a mixed oligopoly. However, they restrict attention to Cournot competition ignoring the effects of private firms' collusion. Our main contribution is twofold. On the one hand, we obtain that collusion incentives are weaker due to the output expanding incentives of the public firm as well as due to the R&D activities. To the best of our knowledge, previous results (see, for instance, Escrihuela-Villar and Gutiérrez-Hita, 2018a) ignored private firms' innovation. One way to interpret this result, which is in line with several empirical studies, is that one could observe an intensification of competition due to the R&D. This is interesting since it is in contrast to the traditional interpretation of the Schumpeterian relation between concentration and R&D. In this context, we also consider partial privatization and show that, even though full privatization always decreases welfare compared to the market with a pure public firm, there is an inverse U-shaped relationship between total welfare and privatization such that total welfare is always maximized at a certain level of partial privatization. Secondly, we obtain that the well-known neutrality result of privatization when an optimal production subsidy is considered is not satisfied when firms innovate. This is true regardless of the degree of collusion between private firms. This result proves useful to identify that some previous results (see, Lin and Matsumura, 2018 or Escrihuela-Villar and Gutiérrez-Hita, 2018b) crucially rely on the assumption that firms do not innovate. Besides, we can also characterize the effects of privatization with a production subsidy to obtain that privatizing is recommended when collusion is low enough. On the contrary, when competition between private firms is weak, a production subsidy should be complemented with a regulation limiting the privatization scheme.

The present paper presents an analysis of the incentives of private firms to collude but does not consider collusion sustainability. In this sense thus, further research is required, incorporating, for instance, a repeated non-cooperative game with tacit collusion. In this line, and using also a relative performance model, Matsumura and Matsushima (2012) have considered the relationship between the degree of competition and the sustainability of the collusive behavior but in a private oligopoly without R&D. This extension might be interesting and opens up potential further research. Indeed, we know from the famous Folk Theorem that any combination of individually rational profits is sustainable if firms are sufficiently patient.

Therefore, our Proposition 1 could help us understand under which circumstances private firms might voluntarily be willing to sustain less collusion in a repeated game. Admittedly, the framework we have worked with is only a particular approach to a more general issue, and further research is surely required. Incorporating price competition, cost asymmetries, firms' capacities or free entry of private firms would probably enrich our analysis. We believe that those are also subjects for future research.

## Appendix

**Proof of Proposition 1.**—As mentioned in Section 2, the intersection of the three reaction functions described in (4) leads us to obtain the equilibrium quantities of the second stage of the game, namely  $q_1(\alpha, \beta, x_1, x_2, x_3, \gamma)$ ,  $q_2(\alpha, \beta, x_1, x_2, x_3, \gamma)$ , and  $q_3(\alpha, \beta, x_1, x_2, x_3, \gamma)$  where for simplicity  $a$  and  $c$  are not included as arguments of these functions. Then, these outputs must be replaced in (3) in order to maximize individual profits with respect to  $x_1$ ,  $x_2$ , and  $x_3$  respectively. We obtain thus  $x_1(\alpha, \beta, \gamma)$ ,  $x_2(\alpha, \beta, \gamma)$ , and  $x_3(\alpha, \beta, \gamma)$  to be replaced in the quantities obtained before which will lead us to the equilibrium quantities  $q_1(\alpha, \beta, \gamma)$ ,  $q_2(\alpha, \beta, \gamma)$ , and  $q_3(\alpha, \beta, \gamma)$ . Firms' profits easily follow:  $\Pi_1(\alpha, \beta, \gamma)$ ,  $\Pi_2(\alpha, \beta, \gamma)$  and  $\Pi_3(\alpha, \beta, \gamma)$ . For simplicity, we assume  $\gamma = 1$  and hence  $\Pi_2(\alpha, \beta, 1) = \Pi_3(\alpha, \beta, 1) \equiv \frac{x}{y}$  where  $x = (a - c)^2(4(\alpha - 3)^2(2 + \alpha) - 18 + 5\beta + (\beta - 4)\alpha^2 - (60 - 16\beta + (\beta - 4)(\alpha - 3)\alpha^2)(88 + 20\alpha + \beta(-59 - 13\alpha + 2\beta(5 + \alpha)))^2$  and where  $y = (51744 - 2\beta(22816 + \beta(-6753 + 670\beta)) + (17040 + \beta(-14112 + (3825 - 338\beta)\beta))\alpha + 4(\beta - 4)(261 + \beta(29\beta - 173))\alpha^2 + (\beta - 4)(552 + \beta(46\beta - 321))\alpha^3 + 2(\beta - 4)^2(2\beta - 7)\alpha^4)^2$ . We have to check thus that private firms' profits are maximized with respect to  $\alpha$  at a value between 0 and 1. Unfortunately, the solution for the equation  $\frac{\partial \Pi_3(\alpha, \beta, 1)}{\partial \alpha} = 0$  for  $\alpha$  cannot be reported in the body text of the paper. We proceed as follows: we define the function  $\frac{\partial \Pi_2(\alpha, \beta, 1)}{\partial \alpha} \equiv f(\alpha, \beta, 1)$  and check that  $f(\alpha, \beta, 1)$  has only one real root at  $\hat{\alpha}$  where  $\hat{\alpha} \in (0, 1)$ . The second-order condition is also satisfied, namely,  $\frac{\partial^2 \Pi_2(\alpha, \beta, 1)}{\partial^2 \alpha} < 0$ . Regarding the second part of the proposition, we just have to apply the implicit function theorem. Then, the relationship between the profit maximizing  $\alpha$  and  $\beta$  can be obtained respectively with the sign of  $-\frac{\frac{\partial f(\alpha, \beta, 1)}{\partial \beta}}{\frac{\partial f(\alpha, \beta, 1)}{\partial \alpha}}$  and  $-\frac{\frac{\partial f(\alpha, \beta, \gamma)}{\partial \gamma}}{\frac{\partial f(\alpha, \beta, \gamma)}{\partial \alpha}}$ . Regarding the first one, if we evaluate  $-\frac{\frac{\partial f(\alpha, \beta, 1)}{\partial \beta}}{\frac{\partial f(\alpha, \beta, 1)}{\partial \alpha}}$  at  $\beta = 1$ , we get  $-\frac{\frac{\partial f(\alpha, \beta, 1)}{\partial \beta}}{\frac{\partial f(\alpha, \beta, 1)}{\partial \alpha}} \equiv \frac{s}{t}$  where  $s = 15618092988 - \alpha(7067813064 + \alpha(14134484001 + \alpha(1401220378 + \alpha(-2767395718 + 3\alpha(-131933156 + \alpha(123274961 + 2\alpha(10651589 + 3\alpha(-1736326 + \alpha(-476618 + 9\alpha(1481 + 12\alpha(127 + 10\alpha))))))))))$  and  $t = (13 + 3\alpha)^2(-443855227 + \alpha(304287508 + 3\alpha(-2729926 + \alpha(-41563404 + \alpha(-1198486 + 3\alpha(774552 + 5\alpha(-30994 + 3\alpha(-3556 + 3\alpha(69 + 20\alpha))))))))))$  and check that  $-\frac{\frac{\partial f(\alpha, \beta, 1)}{\partial \beta}}{\frac{\partial f(\alpha, \beta, 1)}{\partial \alpha}}$  is positive. Also, we just have to check that  $\frac{\partial f(\alpha, 1, \gamma)}{\partial \gamma}$  is also always positive. Finally, we can check that  $\lim_{\gamma \rightarrow \infty} f(\alpha, \beta, \gamma) = \frac{(a - c)^2(2 + \alpha(\beta - 4) - \beta)(\beta - 3)^2}{(-18 + \alpha(\beta - 4) + 5\beta)^3}$  and this function has a root at  $\alpha = \frac{\beta - 2}{\beta - 4}$ . Further details have been omitted from the paper since the expressions could not be simplified and included in the body text of the paper. However, they are available in an additional online appendix<sup>11</sup>.

**Proof of Corollary 1.**—Since private firms' profits have only one maximum with respect to  $\alpha$ , we just have to check that  $\frac{\partial \Pi_i}{\partial \alpha}$ , with  $i = 2, 3$ , evaluated at  $\alpha = \frac{\beta - 2}{\beta - 4}$  is negative. Consequently,  $\hat{\alpha} < \frac{\beta - 2}{\beta - 4}$  if  $\gamma$  is finite. In other words, the first order condition of the private firms'

profits with respect to  $\alpha$  is negative when we evaluate the function at the profit maximizing  $\alpha$  of the allocation where there is no R&D. Therefore, since there is just one maximizing  $\alpha$ , this value is smaller when  $\gamma$  is finite compared to the limit case where  $\gamma$  tends to infinite. This proves Corollary 1.

**Proof of Proposition 2.**—From the definition of welfare used in the present paper,  $W \equiv \sum_{i=1}^3 \Pi_i + CS$  where  $CS = \frac{(q_1(\alpha, \beta, \gamma) + q_2(\alpha, \beta, \gamma) + q_3(\alpha, \beta, \gamma))^2}{2}$ . For the first part, we just have to evaluate  $W$  at  $\beta=1$  to check that its value is larger than at  $\beta=0$ . For the second part, we can check that the equation  $\frac{\partial W}{\partial \beta} = 0$  has only one root at  $\hat{\beta}$  in the interval  $\beta \in (0, 1)$  and, in addition, the second order condition is also satisfied:  $\frac{\partial^2 W}{\partial^2 \beta} < 0$ . Finally, and applying the implicit function theorem, we can define the function  $\frac{\partial W(\alpha, \beta, \gamma)}{\partial \beta} \equiv g(\alpha, \beta, \gamma)$  where  $g(\alpha, \beta, 1) \equiv \frac{s}{q}$  where  $s = 32(-3053207520 + \beta(9724583088 + \beta(-11546515926 + \beta(7075719690 + \beta(-2484918051 + 10\beta(50736147 + 25\beta(-224947 + 10498\beta)))))))(a-c)^2$  and  $q = (-25872 + \beta(22816 + \beta(-6753 + 670\beta)))^3$ . Then,  $\left. \frac{\partial \hat{\beta}}{\partial \alpha} \right|_{\alpha=0} < 0$  is true if  $-\frac{\frac{\partial g(\alpha, \beta, 1)}{\partial \alpha}}{\frac{\partial g(\alpha, \beta, 1)}{\partial \beta}} < 0$  when  $\alpha=0$ . We just have to check thus that  $\frac{\partial g(\alpha, \beta, 1)}{\partial \alpha} < 0$  and  $\frac{\partial g(\alpha, \beta, 1)}{\partial \beta} > 0$  are true if  $\alpha=0$ . Finally, the equation  $\frac{\partial W}{\partial \beta} = 0$  can be explicitly solved at  $\alpha=0$  and  $\alpha=1$  to obtain the different reported values of  $\hat{\beta}$ .

**Proof of Proposition 3.**—We added here an optimal production subsidy  $s$  that maximizes welfare. Consequently, we have a different value of the welfare function than the one obtained in the previous section. We can denote it by  $W^s(\alpha, \beta)$ . Then, we just have to maximize  $W^s(\alpha, \beta)$

with respect to  $\beta$  to check that  $\frac{\partial W^s(\alpha, \beta)}{\partial \beta} < 0$  whenever  $\alpha < \tilde{\alpha}$  while  $\frac{\partial W^s(\alpha, \beta)}{\partial \beta} = 0$  has one positive root at  $\tilde{\beta}$  if  $\alpha \geq \tilde{\alpha}$ , and that the second order condition is satisfied:  $\frac{\partial^2 W^s(\alpha, \beta)}{\partial^2 \beta} < 0$ .

Regarding the second part, if we apply the implicit function theorem, and if we denote

$$\frac{\partial W^s(\alpha, \beta)}{\partial \beta} = 0 \text{ by } h(\alpha, \beta), \text{ we basically have to show that the inequality } -\frac{\frac{\partial h(\alpha, \beta)}{\partial \alpha}}{\frac{\partial h(\alpha, \beta)}{\partial \beta}} > 0 \text{ holds}$$

whenever  $\frac{\partial W^s(\alpha, \beta)}{\partial \beta} = 0$ . The result holds since  $-\frac{\partial h(\alpha, \beta)}{\partial \alpha} > 0$  and  $\frac{\partial h(\alpha, \beta)}{\partial \beta} < 0$ . Furthermore,

$$\frac{\partial W^s(1, \beta)}{\partial \beta} \equiv \frac{m}{n} \text{ where } m \equiv -(4(-660313701376 + 2035512475344\beta - 2253588286368\beta^2 + 851122488248\beta^3 + 425037101256\beta^4 - 635119935459\beta^5 + 330275744216\beta^6 - 96184958040\beta^7 + 16548963489\beta^8 - 1576283670\beta^9 + 64279656\beta^{10})(a-c)^2) \text{ and } n \equiv (91642560 - 218408768\beta + 231940476\beta^2 - 129917556\beta^3 + 39524915\beta^4 - 6177402\beta^5 + 388962\beta^6)^2. \text{ Then, } \frac{m}{n} = 0 \text{ when } \beta \approx 0.83.$$

## Notes

1. These examples include governments increasing or decreasing its ownership of partially privatized firms. This was the case of the Japanese government plans to sell its share in the Japan Post or when the French government increased its ownership of Renault.
2. A more general discussion of this approach can be found in Matsumura *et al.* (2013) and Nakamura (2015). Additionally, Escrihuela-Villar (2015) shows that this formulation and the conjectural variations approach lead to equivalent closed-form solutions.
3. Regarding the quantity competition, this approach is mathematically equivalent to a model of standard strategic delegation in which in the first stage, profit-maximizing firms make a commitment not to maximize their profits while in the second stage, these firms face Cournot competition (see, for instance, Fershtman, 1985, or Vickers, 1985 and subsequent contributions). This literature has mainly reached consensus on the fact that players who care not only about their payoffs but also about their payoffs relative to others earn strictly higher payoffs than do the standard payoff maximizers.
4. See, for instance, Symeonidis (2008) or Matsumura *et al.* (2013).
5. We note also the absence of spillovers. The reason is basically that spillovers dilute the strategic incentive to undertake R&D. Since we focus on the competitive effects of innovation and privatization, the results obtained thus cannot be attributed to the spillover effects but to the pure strategic incentives of firms.
6. Tandon (1984) points out that in a non-strategic model R&D would only be used to minimize costs. Therefore, the equilibrium would be the standard cost-minimization Cournot equilibrium that would naturally arise if R&D and output were simultaneously determined.
7. We thank a referee for raising this point. Admittedly, welfare-improving cooperation between public and private firms might also be formed assuming, for instance, that firms are also concerned with corporate social responsibility (see, for instance, Haraguchi and Matsumura, 2018)
8. Except for the analysis of this limit case, we set throughout the paper  $\gamma=1$  to ensure that the second-order condition of firm  $i$ 's maximization problem is satisfied.
9. An alternative approach consists of private firms choosing  $\alpha$  between stages 1 and 2, namely when firms had already decided upon the R&D. In this case, one could expect that private firms have more incentives to collude. Numerical simulations show, for instance, that  $\hat{\alpha}$  decreases with  $c$ . Roughly speaking; a more efficient firm might find more profitable to reduce production to increase the market price. We thank a referee for bringing this issue to our attention. It represents an interesting avenue to extend the current work, and we left it for future research.
10. We did not consider cooperative R&D in the form of a research joint venture (RJV). The usual driving forces of an RJV formation include manufacturers coming together to solve common manufacturing problems by leveraging resources and sharing risks, among other things, in innovation (see, for instance, Hernán *et al.*, 2003). We believe thus that our assumption is justified in a context where (public and private) firms differ in their objective functions. Assuming an RJV might affect our results as long as the absence of R&D rivalry reduces the negative externality arising from the non-cooperative R&D and increases the incentives to innovate. Presumably, and following the reasoning provided above, private firms' incentives to collude could also be stronger.
11. We used the program Wolfram Mathematica 7.0. Further details for this and subsequent proofs are available at <https://shorturl.at/gqEQY> or from the authors upon request.

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## Resumen

Consideramos un oligopolio mixto donde las empresas deciden las cantidades que producirán y donde las empresas pueden innovar para reducir su coste marginal de producción. Además, las empresas privadas también pueden reducir la competencia en la producción a través de un acuerdo colusorio. En este contexto, obtenemos que los incentivos de colusión son más débiles debido a las actividades de I+D. También investigamos dos medidas reguladoras diferentes; privatización (posiblemente parcial) y un subsidio a la producción. En el último caso, obtenemos que cuando las empresas innovan, el conocido resultado de la neutralidad de la privatización no se cumple. Además, una política adecuada debería incluir una privatización parcial donde la menor competencia entre las empresas privadas exige un esquema de privatización más débil.

*Palabras clave:* competencia imperfecta, oligopolio mixto, privatización parcial, innovación, subsidios.

*Clasificación JEL:* L11, L13, L52, D43.