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Trade-Off Effect of Pay-As-You-Go Public Pension on Economic and Welfare Volatility

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Abstract

Using an overlapping-generations model where heterogeneous individuals choose their own consumption and labor supply for responding to total factor productivity shocks, this paper finds a trade-off effect of pay-as-you-go public pension on macroeconomic stability and welfare volatility. This paper theoretically proves that pay-as-you-go public pension can reduce volatilities of total consumption and social welfare *at the cost of* increasing volatilities of aggregate output, labor supply, and investment. By reducing the exposure of retirement wealth to aggregate shocks, pay-as-you-go public pension can make individuals work and save less in recessions and more in booms.

Keywords: Pay-as-you-go public pension, macroeconomic stability, social welfare volatility.

JEL Classification: H55, E62, E32.

1. Introduction

Rapidly aging population has entailed fiscal unsustainability problem of traditional pay-as-you-go (PAYG) public pension.[1] Among the various reform proposals to fix the problem, privatization of PAYG public pension has drawn the most attention.[2] Since almost all the workers and retirees of an economy are already in the PAYG public pension system of the economy and affected by a change in the system, the impact of PAYG public pension privatization would be neither trivial nor limited to a small part of economy. Since PAYG public pension protects part of individuals' retirement wealth from exposure to aggregate productivity shocks, the degree of individuals' labor-supply and savings responses to the shocks under PAYG public pension is different from the degree of their responses to the same shocks under no PAYG public pension. Nevertheless, how PAYG public pension affects macroeconomic fluctuations is not studied well. This paper investigates the effect of PAYG public pension on economic and social-welfare volatility over the business cycle.

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So far, most of the existing studies on the effects of PAYG public pension examined only steady-state equilibria during and after privatizing PAYG public pension (e. g., Hubbard and Judd, 1987; İmrohoroğlu *et al*., 2003; Fuster *et al*., 2007; Nishiyama and Smetters, 2007). However, steady-state analyses for themselves cannot lead us to understand how PAYG public pension affects the response of an economy to aggregate shocks that divert the economy from its steady state. Although Krueger and Kubler (2006) and Olovsson (2010) conducted simulation analyses by taking four different levels of total factor productivity (TFP), they did not analyze the effect of PAYG public pension on volatility of social welfare or macroeconomic variable. In fact, both adopted overlapping-generations models whose time span for one period is not suitable for real business cycle analysis; that is, Krueger and Kubler (2006) assumed nine-period-lived individuals and Olovsson (2010) assumed three-period-lived individuals. More importantly, Krueger and Kubler (2006) and Olovsson (2010) assumed that individuals' labor supply is fixed (i. e., not chosen), which inevitably entails no labor-supply response to aggregate productivity shocks. Because PAYG public pension benefit and contribution depend on *labor* earnings, for analyzing effect of PAYG public pension on macroeconomic volatility, it is crucial and necessary to allow individuals to choose their own labor supply, instead of assuming fixed labor supply.

In addition, although they are valuable and useful in their own right, simulation studies on the effects of PAYG public pension are inherently limited for the following reasons. Firstly, each simulation result is inevitably case-specific and depends on data of a specific country; hence, the result is not directly applicable to similar reform of another country or other reform of the same country. Secondly, because every step of the computation procedure of a simulation is not explicitly demonstrated, it is not easy for readers to transparently verify whether there is any mistake or error in the procedure or not. Thirdly, no matter how perfectly a simulation is executed, the simulation result in itself does not show the underlying logic and economic intuition. Therefore, overcoming these limitations, theoretical analysis regarding the effect of PAYG public pension on macroeconomic and welfare volatility is as much worthwhile as simulation analysis on it.

To achieve wide applicability and improve upon the previous studies, this paper adopts an overlapping-generations model that allows heterogeneous individuals to choose their own labor supply and consumption for deriving individuals' optimal responses to an aggregate productivity shock. Based on the optimal responses, this paper theoretically proves that PAYG public pension can reduce volatilities of total consumption and social welfare *at the cost of* raising economic volatilities of aggregate labor supply, investment, and output over the business cycle. Basically, since PAYG public pension reduces exposure of individuals' retirement wealth to macroeconomic shocks, their wealth decreases more in recessions and increases more in booms without PAYG public pension than with PAYG public pension. This extra decrease or increase exerts wealth effects on individuals' labor-supply and savings responses to negative and positive aggregate shocks. Consequently, individuals decrease their labor supplies and savings (investment) by a *larger* margin in recessions with PAYG public pension than without PAYG public pension; and, they increase their labor supplies and savings by a larger margin in booms with PAYG public pension than without PAYG public pension. As a result, by increasing volatilities of aggregate labor supply and investment, PAYG public pension raises total output instability. At the same time, since PAYG public pension provides intergenerational risk-sharing that helps individuals to reduce consumption volatility, individuals decrease their consumption by a *smaller* margin in recessions with PAYG public pension than without PAYG public pension, whereas they increase their consumption by a smaller margin in booms with PAYG public pension than without PAYG public pension. While PAYG public pension increases macroeconomic instability, it reduces volatilities of total consumption and social welfare over the business cycle. Individuals suffer less from working less and consuming more in recessions with PAYG public pension than without PAYG public pension, whereas they enjoy less from working more and consuming less in booms with PAYG public pension than without PAYG public pension.

Notably, this paper newly discovers the trade-off effect of PAYG public pension on the aggregate output instability and welfare volatility, which has never been rigorously analyzed despite its importance. The findings of this paper imply that privatization of PAYG public pension can reduce volatilities of aggregate output, labor supply and investment at the cost of increasing instability of consumption and social welfare. Thus, this paper suggests the need to consider the overlooked factor of consequent changes in volatilities of total output and social welfare for accurately evaluating the reform proposal of PAYG public pension privatization. Actually, the volatility of total output, labor supply and investment is one of the key macroeconomic variables and gives rise to various stabilization policies; hence, it is important to realize that PAYG public pension, which is quite sizable,[3] affects the volatility.

The rest of this paper proceeds as follows. Section 2 describes a theoretical model from which optimal response to aggregate productivity shocks is derived in Section 3. Based on this, the effect of PAYG public pension on volatilities of economic variables and social welfare is analysed in Section 4. The last section concludes the paper.

2. The Model

Consider an economy inhabited by heterogeneous individuals who differ in age and earning ability. In each period, individuals are born with zero endowment and live up to the age of I. Thus, I different age cohorts co-exist in the economy. An age-i individual survives to be of age $i+1$ with probability m_i, while the total population is normalized to and stays as one. Moreover, in each period, individuals' earning ability $e \in E \equiv [\underline{e}, \bar{e}]$ is subject to an independent identically distributed idiosyncratic shock.

In each period, for any given $i \in \{1,...,I\}$, an age-i individual chooses his own labor supply and consumption for his remaining life time by solving the following maximization problem:

$$\max_{\{c_{t+h-i,h}, l_{t+h-i,h}\}_{h=i}^{I}} u(c_{t,i}, l_{t,i}) + \sum_{h=i+1}^{I} E\left[\beta^{h-i}\left(\prod_{s=i+1}^{h} m_{s-1}\right)u(c_{t+h-i,h}, l_{t+h-i,h})\right], \quad (1)$$

where $c_{t,i}$ and $l_{t,i}$ are consumption and labor supply, respectively, at age i in period t; β is time preference parameter. The amount of time given to each individual for each period is one; thus, $l_{t,i} \in [0,1]$ for $\forall t$ and $\forall i$. Moreover, the within-period utility function is

$$u(c_{t,i}, l_{t,i}) = log(c_{t,i}) - \psi \frac{l_{t,i}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \quad (2)$$

To be consistent with empirical findings on labor supply behavior (Chetty, 2006), the coefficient of relative risk aversion is set to 1. When solving (1), the following intertemporal budget constraint should be met for each period:

$$w_t e l_{t,i}(1 - \tau_p - \tau_l) + (1 + (1 - \tau_k) r_t) k_{t,i} + q_t + 1_R(i) a_{t,i} \geq c_{t,i} + k_{t+1,i}, \quad (3)$$

where w_t and r_t are market wage rate and interest rate, respectively, of period t; e is realized earning ability; τ_p is PAYG public pension contribution rate, whereas τ_l and τ_k are rates of tax on labor income and capital gain, respectively; q_t is accidental bequest from those who die in period t; and, $k_{t+1,i}$ is investment (savings) made by an age-i individual in period t whose gain returns in the next period (period t+1). Furthermore, $1_R(i)$ indicates whether an age-i individual is eligible to receive PAYG public pension benefit (taking the value of one if he is eligible or the value of zero otherwise) and the amount of PAYG public pension benefit for him in period t is $a_{t,i}$ calculated by a given formula. That is, reflecting the fact that almost all PAYG public pension systems are defined benefit pension plans, the benefit formula is exogenously given and does not depend on financial market returns.[4] For each period, individuals face a borrowing constraint; i.e., $k_{t,i} \geq 0$ for $\forall t$ and $\forall i$. Since the budget constraint (3) is binding for each period, once labor supply and consumption are decided, savings are automatically determined.

Although individuals do not intend to leave any bequest, due to the uncertainty on their longevity, they accidentally leave some bequests. For simplicity, the sum of the bequests that are left accidentally is distributed equally to each surviving individual, as in previous studies like Nishiyama and Smetters (2007). Thus, $\forall t$,

$$q_t = \sum_{i=1}^{I} (1 - m_i) \int_{E\times K\times A} k_{t,i}(\boldsymbol{s}_t)\, dP_t(\boldsymbol{s}_t) \left\{ \sum_{i=1}^{I} m_i \int_{E\times K\times A} dP_t(\boldsymbol{s}_t) \right\}^{-1}, \quad (4)$$

where $\boldsymbol{s}_t = (i, e, k_{t,i}, a_{t,i})$ is a vector of the state variables of period t; the sets of E, K, and A denote the supports for earning ability, wealth (capital), and potential amount of public pension benefit,[5] respectively. The distribution of the state variables $P_t(s_t)$ evolves as follows: $\forall t$,

$$p_{t+1}(i+1, e, k', a') = \\ = \frac{m_i \int_{E\times K\times A} 1(k' = k_{t,i}(\boldsymbol{s}_t) + q_t) \times 1(a' = a_{t+1,i+1}(w_t e l_{t,i}(\boldsymbol{s}_t), a_{t,i})) dP_t(\boldsymbol{s}_t)}{1+n}, \quad (5)$$

where $p_{t+1}(\boldsymbol{s}_{t+1})$ refers to the probability density of $\boldsymbol{s}_{t+1}$; $1(\cdot)$ is a binary indicator function that takes the value of one if statement inside the parenthesis is true and the value of zero otherwise. Moreover, $l_{t,i}(\boldsymbol{s}_t)$ and $k_{t,i}(\boldsymbol{s}_t)$ are decision rules of labor supply and savings (investment), respectively, of an age-i individual in period t. In succint and recursive way, the maximization problem that an age-i individual solves in period t is restated as

$$V_i(\boldsymbol{s}_t) = \max_{\{c_{t,i}, l_{t,i}\}} u(c_{t,i}, l_{t,i}) + m_i \beta E[V_{i+1}(\boldsymbol{s}_{t+1})], \tag{6}$$

where $V_i(\cdot)$ is the maximized utility function (value function) of an age-i individual for the remaining lifetime subject to (3), (4), and (5).

In addition, there exists a representative firm in this economy, which produces output Y_t for period t, following Cobb-Douglas technology: that is, with $1 > \alpha > 0$.

$$Y_t = F(K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{7}$$

where K_t is aggregate capital invested by individuals in the economy and L_t is the sum of individuals' labor supply in efficiency unit. In particular, z_t is total factor productivity (TFP) that is subject to a random shock as

$$z_t = z_{t-1}^{\rho} \, exp(\varepsilon_t) \text{ where } \varepsilon_t \sim WN(0, \sigma_z^2) \text{ for } \forall t. \tag{8}$$

Moreover, the shock on TFP is not correlated with the age structure.[6] The firm solves its profit maximization problem of $\max_{\{K_t, L_t\}} z_t K_t^{\alpha} L_t^{1-\alpha} - (r_t + \delta) K_t - w_t L_t$, which yields the following decision rules: for $\forall t$,

$$F_K(K_t, L_t) = r_t + \delta, \tag{9}$$

$$F_L(K_t, L_t) = w_t, \tag{10}$$

where δ is a given depreciation rate of capital; $F_K(K_t, L_t) = \dfrac{\partial F(K_t, L_t)}{\partial K_t}$ and $F_L(K_t, L_t) = \dfrac{\partial F(K_t, L_t)}{\partial L_t}$.

The government chooses labor income tax rate τ_l to balance its budget as follows.

$$G + \sum_{i=R}^{I} \int_{E \times K \times A} a_{t,i}(\boldsymbol{s}_t) \, dP_t(\boldsymbol{s}_t) = \\ = \sum_{i=1}^{I} \int_{E \times K \times A} \{w_t e l_{t,i}(\boldsymbol{s}_t)[\tau_l + \tau_p] + \tau_k r_t k_{t,i}(\boldsymbol{s}_t)\} dP_t(\boldsymbol{s}_t), \tag{11}$$

where G is a given public expenditure on public goods,[7] and R is PAYG public pension entitlement age at which an individual becomes eligible to receive PAYG public pension benefit. While individuals cannot receive PAYG public pension benefits before the entitlement age R, they do not delay or stop receiving the benefits after the age R.[8] Thus,

$$1_R(i) = \begin{cases} 0 & \text{if } i < R \\ 1 & \text{if } i \geq R \end{cases}. \tag{12}$$

Moreover, this economy as a whole meets the following aggregate resource constraint: for $\forall t$,

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t + G, \tag{13}$$

where $C_t \equiv \sum_{i=1}^{I} \int_{E\times K\times A} c_{t,i}(\boldsymbol{s}_t)\, dP_t(\boldsymbol{s}_t)$ is aggregate consumption. Via competitive markets where individuals and the firm solve their own maximization problems, this economy reaches a stationary general equilibrium that is defined as below.

Given government policies $\{G,R,\tau_p,\tau_k\}$ and the public pension benefit formula, the stationary competitive general equilibrium of this economy is a set of value functions $\{V_i(\boldsymbol{s}_t)\}_{i=1}^{I}$, individuals' decision rules $\{c_{t,i}(\boldsymbol{s}_t), l_{t,i}(\boldsymbol{s}_t), k_{t,i}(\boldsymbol{s}_t)\}_{i=1}^{I}$, the associated distribution of the state variables defined by (5), a lump-sum transfer of accidental bequest q_t, labor income tax rate τ_l, and the factor prices of labor w_t and capital r_t, which satisfies the following conditions for $\forall t$:

(i) Given the government policies, factor prices, and transfer of accidental bequest, all individuals' decision rules solve their own maximization problem of (6).

(ii) The representative firm maximizes its profit by satisfying (9) and (10) with competitive factor markets being cleared as below:

$$K_t = \sum_{i=1}^{I} \int_{E\times K\times A} k_{t,i}(\boldsymbol{s}_t)\, dP_t(\boldsymbol{s}_t), \tag{14}$$

$$L_t = \sum_{i=1}^{I} \int_{E\times K\times A} e l_{t,i}(\boldsymbol{s}_t)\, dP_t(\boldsymbol{s}_t), \tag{15}$$

which satisfy (13) as well.

(iii) The government sets τ_l according to (11) and transfers q_t that is defined by (4).

(iv) This economy reaches a steady state by meeting

$$p_t(\boldsymbol{s}) = p_{t+1}(\boldsymbol{s}) \;\text{ for }\; \forall \boldsymbol{s} \in \{1,\cdots,I\} \times E \times K \times A \;\text{ and }\; \forall t. \tag{16}$$

Once reaching a steady state, the time subscript is not necessary; hence, it is dropped. Moreover, as individuals' utilities are at their own maximum under a stationary general equilibrium, steady-state social welfare (which is the sum of the utilities of all individuals for their remaining life time) is $SW = \sum_{i=1}^{I} \int_{E\times K\times A} V_i(\boldsymbol{s}) dP(\boldsymbol{s})$.

3. Optimal Responses to Total Factor Productivity Shocks

After reaching a steady-state general equilibrium, this economy may fluctuate with shocks on TFP. In particular, a TFP shock propagates throughout the entire economy via responses of individuals and the firm. According to Uhlig (1999), equilibrium laws of their responses to a TFP shock are approximated by log-linearization. From the dynamic optimization of (6), individuals' log-linearized optimal responses of labor supply, consumption, and savings to an aggregate productivity shock are as follows: for $\forall s \in \{1,2,\ldots\}$

$$\frac{1}{\eta}\hat{l}_{s,i} = \hat{Y}_s - \frac{1}{\bar{L}}\left(\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s})\right) - \hat{c}_{s,i} \text{ for } \forall i \in \{1,\ldots,I\} \tag{17}$$

$$\hat{c}_{s,i} = E\left[\hat{Y}_{s+1} - \frac{1}{\bar{K}}\sum_{i=1}^{I-1}\int_{E\times K\times A}\hat{k}_{s+1,i+1}\bar{k}_{i+1}(\boldsymbol{s})dP(\boldsymbol{s}) - \hat{c}_{s+1,i+1}\right] \text{ for } \forall i \in \{1,\ldots,I-1\} \tag{18}$$

$$\begin{aligned}&\left(\hat{Y}_s - \frac{\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s})}{\bar{L}} + \hat{l}_{s,i}\right)\bar{w}e\bar{l}_i(1-\tau_l-\tau_p) + \\ &+(1-\tau_k)\bar{r}\bar{k}_i\left(\hat{Y}_s - \frac{\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}\bar{k}_i(\boldsymbol{s})\,dP(\boldsymbol{s})}{\bar{K}} + \hat{k}_{s,i}\right) + \hat{q}_s\bar{q} + \bar{k}_i\hat{k}_{s,i} = \\ &= \bar{c}_i\hat{c}_{s,i} + \bar{k}_{i+1}\hat{k}_{s+1,i+1} \text{ for } \forall i \in \{1,\ldots,I\},\end{aligned} \tag{19}$$

where s refers to the number of periods that elapse from the moment when a TFP shock is materialized; $\bar{x}$ is steady-state value of variable x_s; and, $\hat{x}_s = log\left(\frac{x_s}{\bar{x}}\right)$. As Uhlig (1999) showed, $100\hat{x}_s \approx 100\frac{(x_s-\bar{x})}{\bar{x}}$ approximates % deviation of the variable x_s from its steady-state value $\bar{x}$ so that volatility (responsiveness) of the variable x_s over the business cycle is measured by averaging $\hat{x}_s$ s from TFP shocks of various values. On the other hand, $\hat{x}_s\bar{x} \approx x_s - \bar{x}$ approximates variable x_s's absolute deviation (absolute distance) from its steady-state value $\bar{x}$ in its responding to a TFP shock. From (7) and (8), the representative firm's optimal response by which a TFP shock spreads throughout the entire economy is as follows:

$$\hat{Y}_s = \hat{z}_s + \frac{\alpha}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}\bar{k}_i(\boldsymbol{s})dP(\boldsymbol{s}) + \frac{(1-\alpha)}{\bar{L}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s}), \tag{20}$$

$$\hat{z}_s = \rho\hat{z}_{s-1} + \varepsilon_s. \tag{21}$$

For details on deriving the optimal responses of (17), (18), (19), (20), and (21), refer to Appendix A. By introducing a TFP shock with an unexpected change in ε_t of (8), which is initially zero at the steady state, and then solving a system of the linear equations of (17), (18), (19), (20), and (21), economic and welfare responses to the exogenous TFP shock are obtained. On the other hand, optimal responses of economy without PAYG public pension to the same TFP shock are obtained in exactly the same way after setting $\tau_p = 0$ and $a_i = 0$ for $\forall i$.

4. Effect of PAYG Public Pension on Macroeconomic and Welfare Volatilities

Based on the above-described optimal responses to a TFP shock of two economies that are identical except for PAYG public pension, the effect of PAYG public pension on volatilities of macroeconomic variables and social welfare can be identified.

Proposition 1.—If PAYG public pension reduces steady-state total labor supply and increases the absolute deviation of total labor supply from its steady-state value in responding to TFP shocks, PAYG public pension raises volatility of total output over the business cycle by increasing volatilities of aggregate labor supply and investment.

Proof. See Appendix B.

At the same time when PAYG public pension increases volatilities of total output, labor supply, and investment, it also reduces volatilities of aggregate consumption and social welfare.

Proposition 2.—If PAYG public pension reduces steady-state total labor supply and increases the absolute deviation of total labor supply from its steady-state value in responding to TFP shocks, PAYG public pension reduces volatilities of total consumption and social welfare over the business cycle while increasing volatility of total output.

Proof. See Appendix C.

In fact, various existing studies (e.g., Hubbard and Judd, 1987; İmrohoroğlu, *et al.*, 2003; Fuster, *et al.*, 2007; Nishiyama and Smetters, 2007) have established that PAYG public pension reduces steady-state total labor supply and investment. Basically, PAYG public pension contributions (payroll taxes) give disincentive for individuals to supply their labor, while PAYG public pension crowds out individuals' savings (investment) as securing a given amount of the resources for post-retirement consumption requires individuals to save less with PAYG public pension than without PAYG public pension. Above all, these various studies showed that the first condition for *Proposition 1* and *2* (PAYG public pension reduces steady-state total labor supply) is not restrictive at all.

In addition, the remaining second condition (PAYG public pension increases the absolute deviation of total labor supply from its steady-state value in responding to TFP shocks) is met when PAYG public pension takes up a large retirement wealth. Notably, PAYG public pension reduces exposure of individuals' retirement wealth to aggregate shocks, whereas individuals fully expose their retirement wealth to aggregate shocks without PAYG public pension. As a result, facing a negative aggregate shock, without PAYG public pension, individuals suffer greater loss in their retirement wealth than with PAYG public pension; and, facing a positive aggregate shock, they enjoy greater gain in their retirement wealth than with PAYG public pension. Thus, for responding to a negative aggregate shock, individuals reduce their labor supply and savings by a *larger* margin with PAYG public pension than without PAYG public pension. For responding to a positive aggregate shock, individuals increase their labor supply and savings by a larger margin with PAYG public pension than without PAYG public pension. When this wealth effect from the reduced exposure to the aggregate shock by PAYG public pension is large enough, post-shock total labor supply deviates from its steady-state value by a larger margin than without PAYG public pension. Thus, with wealth protected by PAYG public pension being large enough to meet this aggregate condition, PAYG public pension increases volatilities of aggregate labor supply and investment (savings). Moreover, since individuals have the three margins of labor supply, consumption, and savings for absorbing an aggregate shock, an increase in the degree of responsiveness of labor-supply and savings margins (due to the reduced exposure of retirement wealth to aggregate shocks by PAYG public pension) leads to a decrease in the degree of responsiveness of consumption margin for absorbing the same aggregate shock. Also, since PAYG public pension enables an aggregate shock to be shared across generations via its intergenerational link[9] for stable post-retirement consumption, PAYG public pension can reduce instability of total consumption.

Consequently, by raising volatilities of aggregate labor supply and investment (which are the only two inputs for total production), PAYG public pension increases volatility of total output while it reduces volatility of aggregate consumption. As the above-described intuition shows, this result fundamentally originates from the reduced exposure of retirement wealth to TPF shocks by PAYG public pension. Noticeably, as shown in (13), output is used not only for consumption but also for investment (savings); so, an increase in total output volatility does not necessarily entail an increase in aggregate consumption volatility.

At the same time, since individuals' utility depends negatively on labor supply and positively on consumption, individuals suffer *less* from less labor and more consumption facing a negative aggregate shock with PAYG public pension than without PAYG public pension; and, individuals enjoy *less* from more labor and less consumption facing a positive aggregate shock with PAYG public pension than without PAYG public pension. As social welfare after a TPF shock is the post-shock population-weighted sum of individuals' utilities, this implies that PAYG public pension reduces social welfare volatility by causing social welfare to go down less facing a negative aggregate shock and to go up less facing a positive aggregate shock.

In applying *Proposition 1* and *2* to the current PAYG public pension system of developed economies, *Proposition 1* and *2* suggest that after privatization of PAYG public pension, macroeconomic volatility will be decreased at the cost of increasing volatilities of social welfare and total consumption if the two generic conditions are met. Firstly, the existing studies show that the first condition is met (e. g., Hubbard and Judd, 1987; İmrohoroğlu, *et al*., 2003; Fuster, *et al*., 2007; Nishiyama and Smetters, 2007). Secondly, from the above-described wealth-effect intuition of *Proposition 1* and *2* (the larger retirement wealth is protected by PAYG public pension from TFP shocks, the more likely the second condition is to be met), the remaining second condition is also quite likely to be met, as the actual amounts of public pension funds and public pension debt are quite large (e. g., on average, public pension funds of OECD economies amount to 19.6% of GDP in 2013 and their public pension debt is projected to be 124.2 % of GDP in 2033). This paper does not impose any assumption on transition process of privatizing PAYG public pension. No consensus about the transition process has been gained either in practice or theory, and Huang, *et al*. (1997) and Kotlikoff, *et al*. (1999) found that welfare cost from the transition process is sensitive to various assumptions on the transition process. Nevertheless, various previous studies (e. g., Hubbard and Judd, 1987; Huang, *et al*., 1997; Kotlikoff, *et al*., 1999; İmrohoroğlu, *et al*. 2003; Fuster, *et al*., 2007; Nishiyama and Smetters, 2007) have shown that a steady-state equilibrium reached after finishing privatization of PAYG public pension is *independent* of the transition process. Therefore, the theoretical findings of *Proposition 1* and *2* are also independent of the transition process, since they deal with macroeconomic fluctuations from a steady-state equilibrium after finishing privatization of PAYG public pension.

In short, this paper discovers a previously overlooked trade-off effect of PAYG public pension: reduction in volatility of social welfare *at the cost of* an increase in macroeconomic instability. Since the ultimate criteria for evaluating a policy reform (such as privatizing PAYG public pension) is social welfare, improvement in economic stability from policy reform (e. g., from privatizing PAYG public pension) might not be desirable if it does not end up with improving social welfare, although detailed analysis on how to incorporate changes in welfare volatility and economic volatility into the overall evaluation of social welfare are beyond the scope of this paper.

5. Concluding Remarks

This paper investigates the impact of PAYG public pension on economic and welfare volatility, utilizing an overlapping-generations model where heterogeneous individuals choose their own labor supply and consumption. This paper theoretically proves that PAYG public pension can cause total output to fluctuate more (i. e., less stable) by raising volatilities of aggregate labor supply and investment, while reducing volatilities of social welfare and total consumption over the business cycle. Notably, this study discovers a previously overlooked trade-off effect of PAYG public pension –reducing social welfare volatility *at the cost of* increasing macroeconomic instability.

Appendix

A. Derivation of Log-linearized Optimal Responses to an Aggregate Productivity Shock

The optimal responses to a TFP shock are approximated following the standard log-linearization procedure of Uhlig (1999). Around the steady state of an economy, $100\%\hat{x}$ deviation from the steady-state value $\bar{x}$ is formulated in terms of after-shock value x_s as $\hat{x}_s = log\left(\frac{x_s}{\bar{x}}\right)$ where *s* refers to the number of periods that elapse from the moment when a TFP shock is materialized; and, $\bar{x}$ is steady-state value of variable x_s. Then,

$$x_s = \bar{x}\, exp(\hat{x}_s) \approx \bar{x}(1 + \hat{x}_s) \text{ and } \hat{x}_s \approx \frac{x_s - \bar{x}}{\bar{x}} \tag{A1}$$

At the optimum of (6), individuals maximize their own utility of (1) subject to the budget constraint of (3). For any given age *i* and period *s*, the intra-temporal optimality condition of age-*i* individuals is

$$\psi l_{s,i}^{\frac{1}{\eta}} = \frac{1}{c_{s,i}} w_s e(1 - \tau_p - \tau_l) = \frac{1}{c_{s,i}}(1-\alpha)\frac{Y_s}{L_s} e(1 - \tau_p - \tau_l) \tag{A2}$$

due to (7) and (10). At the steady state of this economy,

$$\psi \bar{l}_i^{\frac{1}{\eta}} = \frac{1}{\bar{c}_i}(1-\alpha)\frac{\bar{Y}}{\bar{L}} e(1 - \tau_p - \tau_l) \tag{A3}$$

Based on (A1), the intra-temporal optimality condition of (A2) is re-stated as

$$\psi\{\bar{l}_i\, exp(\hat{l}_{s,i})\}^{\frac{1}{\eta}} = \psi \bar{l}_i^{\frac{1}{\eta}}\, exp(\frac{1}{\eta}\hat{l}_{s,i}) = \frac{1}{exp(\hat{c}_{s,i})\bar{c}_i}(1-\alpha)\frac{\bar{Y}\, exp(\hat{Y}_s)}{\bar{L}\, exp(\hat{L}_s)} e(1 - \tau_p - \tau_l). \tag{A4}$$

By dividing (A4) with (A3), we get

$$exp(\frac{1}{\eta}\hat{l}_{s,i}) = \frac{1}{exp(\hat{c}_{s,i})}\frac{exp(\hat{Y}_s)}{exp(\hat{L}_s)} \tag{A5}$$

which implies that

$$\frac{1}{\eta}\hat{l}_{s,i} = \hat{Y}_s - \hat{L}_s - \hat{c}_{s,i}. \tag{A6}$$

In addition, according to (15) and (A1),

$$\begin{aligned} \hat{L}_s &\approx \frac{\sum_{i=1}^{I}\int_{E\times K\times A} e l_{s,i}(\boldsymbol{s})dP(\boldsymbol{s}) - \sum_{i=1}^{I}\int_{E\times K\times A} e\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s})}{\bar{L}} = \\ &= \frac{\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s})}{\bar{L}}. \end{aligned} \tag{A7}$$

Therefore, from (A6) and (A7), we obtain (17).

By the same logic, at the optimum of (6), for any given age *i* and period *s*, the following inter-temporal optimality condition of age-*i* individuals is met.

$$\frac{1}{c_{s,i}} = m_i \beta E\left[(1 + r_{s+1})\frac{1}{c_{s+1,i+1}}\right] = m_i \beta E\left[\left(1 + \alpha\frac{Y_{s+1}}{K_{s+1}} - \delta\right)\frac{1}{c_{s+1,i+1}}\right]. \tag{A8}$$

Based on (A1), the inter-temporal optimality condition of (A8) is re-stated as

$$\frac{1}{\bar{c}_i\, exp(\,\hat{c}_{s,i})} = m_i \beta E\left[\left(1 + \alpha\frac{\bar{Y}\, exp(\,\hat{Y}_{s+1})}{\bar{K}\, exp(\,\hat{K}_{s+1})} - \delta\right)\frac{1}{\bar{c}_{i+1}\, exp(\,\hat{c}_{s+1,i+1})}\right]. \tag{A9}$$

Because the deviation of $\hat{x}_s$ is close to zero around the steady state of the economy, $exp(\,\hat{x}_s)$ ≈ 1 which implies that $exp(\,\hat{Y}_{s+1} - \hat{K}_{s+1}) \approx 1$. Based on this, $\left(1 + \alpha\frac{\bar{Y}\, exp(\,\hat{Y}_{s+1})}{\bar{K}\, exp(\,\hat{K}_{s+1})} - \delta\right) \approx$ $\left(1 + \alpha\frac{\bar{Y}}{\bar{K}} - \delta\right) exp(\,\hat{K}_{s+1} - \hat{Y}_{s+1})$. Dividing the approximated (A9) with the steady-state value of (A8) yields

$$\hat{c}_{s,i} = E[\hat{Y}_{s+1} - \hat{K}_{s+1} - \hat{c}_{s+1,i+1}]. \tag{A10}$$

By the same way we identify (A7), $\hat{K}_{s+1} = \frac{1}{\bar{K}}\sum_{i=1}^{I-1}\int_{E\times K\times A} \hat{k}_{s+1,i+1}\bar{k}_{i+1}(\boldsymbol{s})dP(\boldsymbol{s})$ so that we obtain (18).

After a TFP shock, for any given age *i* and period *s*, the budget constraint of (3) is re-stated as

$$\begin{gathered}\bar{w}(1 + \hat{w}_s)e\bar{l}_i(1 + \hat{l}_{s,i})(1 - \tau_p - \tau_l) + (1 + (1 - \tau_k)\bar{r}(1 + \hat{r}_s))\bar{k}_i(1 + \hat{k}_{s,i}) + \\ +\bar{q}(1 + \hat{q}_s) + 1_R(i)a_{t,i} = \bar{c}(1 + \hat{c}_{s,i}) + \bar{k}_{s+1}(1 + \hat{k}_{s+1,i+1}).\end{gathered} \tag{A11}$$

At the steady state of this economy, the budget constraint of (3) is

$$\bar{w}e\bar{l}_i(1 - \tau_p - \tau_l) + (1 + (1 - \tau_k)\bar{r})\bar{k}_i + \bar{q} + 1_R(i)\bar{a}_i = \bar{c}_i + \bar{k}_{i+1}. \tag{A12}$$

Since $\hat{w}_s\hat{l}_{s,i} \approx 0$ and $\bar{r}\bar{k}_i\hat{r}_{s,i}\hat{k}_{s,i} \approx 0$ for any given age *i* and period *s*, subtracting (A12) from (A11) yields

$$\begin{gathered}(\hat{w}_s + \hat{l}_{s,i})\bar{w}e\bar{l}_i(1 - \tau_l - \tau_p) + (1 - \tau_k)\bar{r}\bar{k}_i(\hat{r}_s + \hat{k}_{s,i}) + \hat{q}_s\bar{q} + \bar{k}_i\hat{k}_{s,i} = \\ = \bar{c}_i\hat{c}_{s,i} + \bar{k}_{i+1}\hat{k}_{s+1,i+1}.\end{gathered} \tag{A13}$$

Moreover, according to (7) and (10), for any given period s, $w_s = (1-\alpha)\frac{Y_s}{L_s}$ which is in turn restated as

$$\bar{w}\, exp(\hat{w}_s) = (1-\alpha)\frac{\bar{Y}\, exp(\hat{Y}_s)}{\bar{L}\, exp(\hat{L}_s)}. \tag{A14}$$

At the steady state of this economy,

$$\bar{w} = (1-\alpha)\frac{\bar{Y}}{\bar{L}}. \tag{A15}$$

Dividing (A14) with (A15) entails that $exp(\hat{w}_s) = exp(\hat{Y}_s - \hat{L}_s)$. Therefore,

$$\hat{w}_s = \hat{Y}_s - \hat{L}_s = \hat{Y}_s - \frac{\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s})}{\bar{L}}. \tag{A16}$$

By the same logic, from (7) and (9), we get

$$\hat{r}_s = \hat{Y}_s - \hat{K}_s = \hat{Y}_s - \frac{1}{\bar{K}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}\bar{k}_i(\boldsymbol{s})\, dP(\boldsymbol{s})\,. \tag{A17}$$

Then, plugging (A16) and (A17) into (A13) results in (19).

Based on (7) and (A1), after a TFP shock, for any given period s, $Y_s = z_s K_s^{\alpha} L_s^{1-\alpha}$ which is restated as

$$\bar{Y}\, exp(\hat{Y}_s) = exp(\hat{z}_s)\{\bar{K}\, exp(\hat{K}_s)\}^{\alpha}\{\bar{L}\, exp(\hat{L}_s)\}^{1-\alpha}. \tag{A18}$$

Dividing (A18) with the steady-state output, $exp(\hat{Y}_s) = exp(\hat{z}_s + \alpha\hat{K}_s + (1-\alpha)\hat{L}_s)$. Hence,

$$\hat{Y}_s = \hat{z}_s + \alpha\hat{K}_s + (1-\alpha)\hat{L}_s = $$

$$= \hat{z}_s + \frac{\alpha\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}\bar{k}_i(\boldsymbol{s})dP(\boldsymbol{s})}{\bar{K}} + \frac{(1-\alpha)\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}\bar{l}_i(\boldsymbol{s})dP(\boldsymbol{s})}{\bar{L}}. \tag{A19}$$

According to (8) and (A1), after a TFP shock, for any given period s, $z_s = z_{s-1}^{\rho}\, exp(\varepsilon_s)$ which is restated as

$$exp(\hat{z}_s) = \{exp(\hat{z}_{s-1})\}^{\rho}\, exp(\varepsilon_s) \tag{A20}$$

since $\bar{z} = 1$. (A20) implies that $exp(\hat{z}_s) = exp(\rho\hat{z}_{s-1} + \varepsilon_s)$. Thus, (21) holds.

B. Proof for Proposition 1

[step 0] Consider two economies that are characterized by the model elaborated in Section II and identical except for PAYG public pension. For notational convenience, one economy with PAYG public pension is indicated by superscript p and the other economy without PAYG public pension by superscript np. Suppose that PAYG public pension reduces steady-state total labor supply. That is,

$$\bar{L}^{np} - \bar{L}^{p} > 0. \tag{B1}$$

In addition, also suppose that PAYG public pension increases the absolute deviation of total labor supply from its steady-state value in responding to the same TFP shocks. Then, for $\hat{z}_s^{np} = \hat{z}_s^{p} \neq 0$ with an arbitrarily given s for an arbitrarily given TFP shock that equally hits both economies, the absolute deviation of total labor supply from its steady-state value in responding to the same TFP shock is greater with PAYG public pension than without PAYG public pension, which is stated as follows, according to (A1),

$$\sum_{i=1}^{I} \int_{E\times K\times A} e\, \hat{l}_{s,i}^{np} \bar{l}_i^{np}(\boldsymbol{s}) dP(\boldsymbol{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e\, \hat{l}_{s,i}^{p} \bar{l}_i^{p}(\boldsymbol{s}) dP(\boldsymbol{s}) < 0. \tag{B2}$$

[step 1] Notice from (A7) that volatility of total labor supply is $\hat{L}_s = \frac{1}{\bar{L}} \sum_{i=1}^{I} \int_{E\times K\times A} e \hat{l}_{s,i}$ $\bar{l}_i(\boldsymbol{s}) dP(\boldsymbol{s})$. Thus, (B1) and (B2) from the above step 0 imply that

$$\frac{1}{\bar{L}^{np}} \sum_{i=1}^{I} \int_{E\times K\times A} e\, \hat{l}_{s,i}^{np} \bar{l}_i^{np}(\boldsymbol{s}) dP(\boldsymbol{s}) - \frac{1}{\bar{L}^{p}} \sum_{i=1}^{I} \int_{E\times K\times A} e\, \hat{l}_{s,i}^{p} \bar{l}_i^{p}(\boldsymbol{s}) dP(\boldsymbol{s}) < 0. \tag{B3}$$

That is, with (B1) and (B2), PAYG public pension increases volatility of total labor supply. Moreover, as $\bar{L}^{np} - \bar{L}^{p} = \sum_{i=1}^{I} \int_{E\times K\times A} e\, \bar{l}_i^{np}(\boldsymbol{s}) dP(\boldsymbol{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} e \bar{l}_i^{p}(\boldsymbol{s})\, dP(\boldsymbol{s}) > 0$, (B1) and (B3) implies

$$\sum_{i=1}^{I} \int_{E\times K\times A} \hat{l}_{s,i}^{np}(\boldsymbol{s}) dP(\boldsymbol{s}) - \sum_{i=1}^{I} \int_{E\times K\times A} \hat{l}_{s,i}^{p}(\boldsymbol{s}) dP(\boldsymbol{s}) < 0. \tag{B4}$$

[step 2] To show that PAYG public pension increases volatility of aggregate investment $\widehat{K}_s$ by way of contradiction, suppose not; that is,

$$\frac{1}{\bar{K}^{np}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{s,i}^{np}\, \bar{k}_i^{np}(\boldsymbol{s}) dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{s,i}^{p}\, \bar{k}_i^{p}(\boldsymbol{s}) dP(\boldsymbol{s}) \geq 0. \tag{B5}$$

After aggregating (18) over the same population of the two economies, comparing the aggregated (18) of the economy with PAYG public pension and of the economy without PAYG public pension yields $\left\{\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,dP(\boldsymbol{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{p}\,dP(\boldsymbol{s})\right\}+E\left[\left\{\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s+1,i+1}^{np}\,dP(\boldsymbol{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s+1,i+1}^{p}dP(\boldsymbol{s})\right\}\right]=E\left[\hat{Y}_{s+1}^{np}-\hat{Y}_{s+1}^{p}-\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I-1}\int_{E\times K\times A}\hat{k}_{s+1,i+1}^{np}\,\bar{k}_{i+1}^{np}(\boldsymbol{s})dP(\boldsymbol{s})-\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I-1}\int_{E\times K\times A}\hat{k}_{s+1,i+1}^{p}\,\bar{k}_{i+1}^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\}\right]$. By lagging this equation one period, we obtain

$$\left\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s-1,i-1}^{np}\,dP(\boldsymbol{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s-1,i-1}^{p}\,dP(\boldsymbol{s})\right\}+$$

$$+\left\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,dP(\boldsymbol{s})-\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s,i}^{p}\,dP(\boldsymbol{s})\right\}= \tag{B6}$$

$$=\hat{Y}_{s}^{np}-\hat{Y}_{s}^{p}-\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{np}\,\bar{k}_{i}^{np}(\boldsymbol{s})dP(\boldsymbol{s})-\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{p}\,\bar{k}_{i}^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\}.$$

Similarly, after aggregating (17) over the same population of the two economies, comparing aggregated (17) of the economy with PAYG public pension and of the economy without PAYG public pension yields

$$\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,(\boldsymbol{s})dP(\boldsymbol{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{p}\,(\boldsymbol{s})\,dP(\boldsymbol{s})=$$

$$=\hat{Y}_{s}^{np}-\hat{Y}_{s}^{p}-\left\{\frac{1}{\overline{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}e\,\hat{l}_{s,i}^{np}\bar{l}_{i}^{np}(\boldsymbol{s})dP(\boldsymbol{s})-\frac{1}{\overline{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\,\hat{l}_{s,i}^{p}\bar{l}_{i}^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\}- \tag{B7}$$

$$-\frac{1}{\eta}\left\{\sum_{i=1}^{I}\int_{E\times K\times A}\hat{l}_{s,i}^{np}(\boldsymbol{s})\,dP(\boldsymbol{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{l}_{s,i}^{p}\,(\boldsymbol{s})dP(\boldsymbol{s})\right\}.$$

In addition, after aggregating (20) over the same population of the two economies, comparing aggregated (20) of the economy with PAYG public pension and of the economy without PAYG public pension yields

$$\hat{Y}_s^{np} - \hat{Y}_s^{p} = \alpha\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{np}\,\bar{k}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{p}\bar{k}_i^{p}\,(\boldsymbol{s})dP(\boldsymbol{s})\right\} + \\ + (1-\alpha)\left\{\frac{1}{\overline{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}_{s,i}^{np}\overline{l}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\overline{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}_{s,i}^{p}\,\overline{l}_i^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\}, \tag{B8}$$

as $\hat{z}_s^{np} = \hat{z}_s^{p}$.

By combining (B7) and (B8);

$$\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,(\boldsymbol{s})dP(\boldsymbol{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{p}(\boldsymbol{s})\,dP(\boldsymbol{s}) = \\ = \alpha\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{np}\,\bar{k}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{p}\,\bar{k}_i^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\} + \\ + (-\alpha)\left\{\frac{1}{\overline{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}^{np}\,\overline{l}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\overline{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}^{p}\,\overline{l}_i^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\} + \\ + \left(-\frac{1}{\eta}\right)\left\{\sum_{i=1}^{I}\int_{E\times K\times A}\hat{l}_{s,i}^{np}\,(\boldsymbol{s})dP(\boldsymbol{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{l}_{s,i}^{p}(\boldsymbol{s})\,dP(\boldsymbol{s})\right\}. \tag{B9}$$

Notice that the first bracket of the right hand side of (B9) is positive due to the assumption of (B5) and $\alpha > 0$. Secondly, because of (B3), (B4), $\alpha > 0$, and $\frac{1}{\eta} > 0$, the remaining two brackets of the right hand side of (B9) are strictly positive. Thus, the left hand side of (B9), $\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,(\boldsymbol{s})\,dP(\boldsymbol{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{p}(\boldsymbol{s})\,dP(\boldsymbol{s})$, should be strictly positive, as its right hand side. As this holds for an arbitrarily given *s*, one-lagged term of the left hand side of (B9) is also strictly positive. Therefore,

$$\left\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s-1,i-1}^{np}\,dP(\boldsymbol{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s-1,i-1}^{p}\,dP(\boldsymbol{s})\right\} + \\ + \left\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,dP(\boldsymbol{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}_{s,i}^{p}\,dP(\boldsymbol{s})\right\} > 0 \tag{B10}$$

with $\hat{c}^{p}_{s-1,0} = \hat{c}^{np}_{s-1,0} = 0 = \hat{c}^{p}_{s-1,I+1} = \hat{c}^{np}_{s-1,I+1}$ since age *i* starts from 1 and does not exceed *I*. Moreover, combining (B6) and (B8) yields

$$\left\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{np}_{s-1,i-1}\,dP(\mathbf{s}) - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{p}_{s-1,i-1}\,dP(\mathbf{s})\right\} + \left\{\sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{np}_{s,i}\,dP(\mathbf{s}) - \right.$$
$$\left. - \sum_{i=1}^{I+1}\int_{E\times K\times A}\hat{c}^{p}_{s,i}\,dP(\mathbf{s})\right\} = (\alpha-1)\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{np}_{s,i}\,\bar{k}^{np}_{i}(\mathbf{s})dP(\mathbf{s}) - \right. \tag{B11}$$
$$\left. -\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{p}_{s,i}\bar{k}^{p}_{i}(\mathbf{s})dP(\mathbf{s})\right\} + (1-\alpha)\left\{\frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}^{np}_{s,i}\bar{l}^{np}_{i}(\mathbf{s})dP(\mathbf{s}) - \right.$$
$$\left. -\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\,\hat{l}^{p}_{s,i}\bar{l}^{p}_{i}(\mathbf{s})\,dP(\mathbf{s})\right\}.$$

Since the left hand side of (B11) is strictly positive due to (B10), the right hand side of (B11) must be strictly positive as well. Since $1 > \alpha > 0$, the assumption of (B5) implies that the first bracket of the right hand side of (B11) is negative. Moreover, because of (B3) the second bracket of the right hand side of (B11) is strictly negative. Thus, the right hand side of (B11) is strictly negative: a contradiction from (B5), which proves that

$$\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{np}_{s,i}\,\bar{k}^{np}_{i}(\mathbf{s})dP(\mathbf{s}) - \frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{p}_{s,i}\,\bar{k}^{p}_{i}(\mathbf{s})dP(\mathbf{s}) < 0. \tag{B12}$$

Therefore, PAYG public pension increases volatility of aggregate investment over the business cycle.

[step 3] Due (B8) and $1 > \alpha > 0$, (B3) and (B12) from the above step 1 and 2 imply that

$$\hat{Y}^{np}_{s} - \hat{Y}^{p}_{s} < 0. \tag{B13}$$

Thus, PAYG public pension increases volatility of total output over the business cycle by increasing volatilities of aggregate labor supply and investment.

C. Proof for Proposition 2

[step 0] At a steady-state equilibrium before any TFP shock is materialized, individuals' utilities are at their maximum; hence, steady-state social welfare is

$$\sum_{i=1}^{I}\int_{E\times K\times A}V_i(\mathbf{s})dP(\mathbf{s}) = \sum_{i=1}^{I}\int_{E\times K\times A}u(\bar{c}_i,\bar{l}_i) + m_i\beta E[V_{i+1}(\mathbf{s})]dP(\mathbf{s}). \tag{C1}$$

When a TFP shock hits, utility responses of individuals are realized as a result of deviations of their own *current* labor supply and consumption from their own steady-state levels. Thus, change in their utility brought by a TFP shock is

$$u(c_{s,i}, l_{s,i}) + m_i \beta E[V_{i+1}(\boldsymbol{s})] - \{u(\bar{c}_i, \bar{l}_i) + m_i \beta E[V_{i+1}(\boldsymbol{s})]\} = u(c_{s,i}, l_{s,i}) - u(\bar{c}_i, \bar{l}_i). \quad \text{(C2)}$$

Hence, the difference between post-shock social welfare and steady-state social welfare is $\sum_{i=1}^{I} \int_{E\times K\times A} [u(c_{s,i}, l_{s,i}) - u(\bar{c}_i, \bar{l}_i)] dP(\boldsymbol{s})$. Thus, taking into account the case where steady-state social welfare can take a negative value, volatility of social welfare is

$$s\widehat{w}_s = \sum_{i=1}^{I} \int_{E\times K\times A} \hat{v}_{s,i}\, dP(\boldsymbol{s}) \times sign\left(\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\boldsymbol{s})\, dP(\boldsymbol{s})\right) \quad \text{(C3)}$$

where $\hat{v}_{s,i} = \dfrac{u(c_{s,i}, l_{s,i}) - u(\bar{c}_i, l_i)}{\sum_{i=1}^{I} \int_{E\times K\times A} V_i(\boldsymbol{s}) dP(\boldsymbol{s})}$ and $sign(x) = \dfrac{x}{|x|}$.

[step 1] Consider two economies that are described by the model elaborated in Section II and are identical except for PAYG public pension. For notational convenience, one economy with PAYG public pension is indicated by superscript *p* and the other economy without PAYG public pension by superscript *np*. Suppose that PAYG public pension reduces steady-state aggregate labor supply and increases the absolute deviation of total labor supply from its steady-state value in responding to TFP shocks; i.e., (B1) and (B2) hold for $\hat{z}_s = \hat{z}_s^p \neq 0$ with an arbitrarily given *s* from any given TFP shock that equally hits both economies. Then, according to *Proposition 1*, PAYG public pension increases volatilities of aggregate labor supply and investment; that is, (B3) and (B12) hold. Moreover, from the step 1 of the proof for *Proposition 1*, (B4) also holds. First of all, as shown in the step 3 of the proof for *Proposition 1*, (B3) and (B12) imply (B13) that PAYG public pension increases volatility of total output over the business cycle.

[step 2] From (18) and (20) with lagging one period,

$$\hat{c}_{s,i} + \hat{c}_{s-1,i-1} =$$

$$= (\alpha - 1)\left\{\frac{1}{\bar{K}} \sum_{i=1}^{I-1} \int_{E\times K\times A} \hat{k}_{s,i} \bar{k}_i(\boldsymbol{s}) dP(\boldsymbol{s}) - \frac{1}{\bar{L}} \sum_{i=1}^{I} \int_{E\times K\times A} e\hat{l}_{s,i} \bar{l}_i(\boldsymbol{s}) dP\,(\boldsymbol{s})\right\}. \quad \text{(C4)}$$

By comparing (C4) for the economy with PAYG public pension and for the economy without PAYG public pension,

$$(\hat{c}_{s,i}^{np} - \hat{c}_{s,i}^{p}) + (\hat{c}_{s-1,i-1}^{np} - \hat{c}_{s-1,i-1}^{p}) =$$

$$= (\alpha - 1)\left[\left\{\frac{1}{\bar{K}^{np}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{s,i}^{np}\, \bar{k}_i^{np}(\boldsymbol{s}) dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}} \sum_{i=1}^{I} \int_{E\times K\times A} \hat{k}_{s,i}^{p}\, \bar{k}_i^{p}(\boldsymbol{s})\, dP(\boldsymbol{s})\right\} - \right. \quad \text{(C5)}$$

$$-\left\{\frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}^{np}\bar{l}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}_{s,i}^{p}\,\bar{l}_i^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\}\Bigg].$$

As (C4) and (C5) hold for an arbitrarily given s, the sign of $\hat{c}_{s,i}^{np} - \hat{c}_{s,i}^{p}$ is not different from the sign of $\hat{c}_{s-1,i-1}^{np} - \hat{c}_{s-1,i-1}^{p}$. Thus, the sign of $\hat{c}_{s,i}^{np} - \hat{c}_{s,i}^{p}$ is the sign of the right hand side of (C5). To show that the sign of the right hand side of (C5) is strictly positive by way of contradiction, suppose not; that is, let

$$\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{np}\,\bar{k}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{p}\,\bar{k}_i^{p}(\boldsymbol{s})\,dP(\boldsymbol{s})\right\} - \tag{C6}$$
$$-\left\{\frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}_{s,i}^{np}\bar{l}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}_{s,i}^{p}\bar{l}_i^{p}(\boldsymbol{s})\;dP(\boldsymbol{s})\right\} \geq 0$$

as $(\alpha - 1) < 0$ due to $1 > \alpha > 0$. From (B7) and (B8), we obtain

$$\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{p}(\boldsymbol{s})\,dP(\boldsymbol{s}) =$$

$$= \alpha\left[\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{np}\,\bar{k}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{p}\bar{k}_i^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\} - \right. \tag{C7}$$
$$\left. -\left\{\frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}_{s,i}^{np}\bar{l}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}_{s,i}^{p}\,\bar{l}_i^{p}(\boldsymbol{s})dP(\boldsymbol{s})\right\}\right] -$$
$$-\frac{1}{\eta}\left\{\sum_{i=1}^{I}\int_{E\times K\times A}\hat{l}_{s,i}^{np}(\boldsymbol{s})\,dP(\boldsymbol{s}) - \sum_{i=1}^{I}\int_{E\times K\times A}\hat{l}_{s,i}^{p}(\boldsymbol{s})\,dP(\boldsymbol{s})\right\}.$$

Firstly, due to (C6), the sign of the first bracket of the right hand side of (C7) is positive. Secondly, because $\frac{1}{\eta} > 0$ and (B4), the remaining term of the right hand side of (C7) is strictly positive. Thus, the right hand side of (C7) is strictly positive, which means that the left hand side of (C7) also should be strictly positive. In addition, (C6) and (C5) also imply that the sign of $\hat{c}_{s,i}^{np} - \hat{c}_{s,i}^{p}$ is negative for an arbitrarily given i; hence, its aggregation over the population, which is equal to the left hand side of (C7), is also negative. This is a contradiction from (C6). Hence, the sign of the right hand side of (C5) is strictly positive; i.e.,

$$\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{np}\,\bar{k}_i^{np}(\boldsymbol{s})dP(\boldsymbol{s})-\frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}_{s,i}^{p}\,\bar{k}_i^{p}(\boldsymbol{s})\,dP(\boldsymbol{s})\right\}-$$
$$-\left\{\frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}e\hat{l}_{s,i}^{np}\bar{l}_i^{np}(\boldsymbol{s})dP\,(\boldsymbol{s})-\frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}e\,\hat{l}_{s,i}^{p}\bar{l}_i^{p}(\boldsymbol{s})\;dP(\boldsymbol{s})\right\}<0. \tag{C8}$$

Firstly, (C5) and (C8) imply that for any given *i*,

$$\hat{c}_{s,i}^{np}-\hat{c}_{s,i}^{p}>0. \tag{C9}$$

Secondly, aggregating (C9) over the same population yields

$$\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{np}\,(\boldsymbol{s})dP(\boldsymbol{s})-\sum_{i=1}^{I}\int_{E\times K\times A}\hat{c}_{s,i}^{p}\,(\boldsymbol{s})\,dP(\boldsymbol{s})>0. \tag{C10}$$

Thus, PAYG public pension reduces volatility of aggregate consumption.

[step 3] Whether a decrease in individual consumption volatility brought by PAYG public pension, which is shown in (C9) of the above step 2, decreases volatility of social welfare or not is shown by examining the sign of $\frac{ds\widehat{w}_s}{d\hat{c}_{s,i}}$ for arbitrarily given *s* and *i*. Note that

$$\frac{ds\widehat{w}_s}{d\hat{c}_{s,i}}=\frac{ds\widehat{w}_s}{d\hat{v}_{s,i}}\frac{d\hat{v}_{s,i}}{dc_{s,i}}\frac{dc_{s,i}}{d\hat{c}_{s,i}}. \tag{C11}$$

Since $\frac{d\hat{v}_{s,i}}{dc_{s,i}}\frac{dc_{s,i}}{d\hat{c}_{s,i}}=\frac{u_c\bar{c}_i}{\sum_{i=1}^{I}\int_{E\times K\times A}V_i(\boldsymbol{s})dP(\boldsymbol{s})}$, $\bar{c}_i>0$ and $u_c>0$,

$$sign\left(\frac{d\hat{v}_{s,i}}{dc_{s,i}}\frac{dc_{s,i}}{d\hat{c}_{s,i}}\right)=sign\left(\sum_{i=1}^{I}\int_{E\times K\times A}V_i(\boldsymbol{s})\,dP(\boldsymbol{s})\right). \tag{C12}$$

Secondly, from the above step 0,

$$sign\left(\frac{ds\widehat{w}_s}{d\hat{v}_{s,i}}\right)=sign\left(\sum_{i=1}^{I}\int_{E\times K\times A}V_i(\boldsymbol{s})\,dP(\boldsymbol{s})\right). \tag{C13}$$

From (C11), (C12), and (C13),

$$sign\left(\frac{ds\widehat{w}_s}{d\hat{c}_{s,i}}\right)=sign\left(\sum_{i=1}^{I}\int_{E\times K\times A}V_i(\boldsymbol{s})\,dP(\boldsymbol{s})\right)\times \tag{C14}$$

$$\times\, sign\left(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\boldsymbol{s})dP(\boldsymbol{s})\right) > 0.$$

When aggregating changes in social welfare from differences in individual consumption volatility which are caused by PAYG public pension, (C9) and (C14) imply that decreased consumption volatility due to PAYG public pension reduces volatility of social welfare.

[step 4] From the above step 2, it follows that $\hat{l}^{p}_{s,i} - \hat{l}_{s,i} < 0$ for any given *i*. To show this, comparing (17) for the economy with PAYG public pension and the economy without PAYG public pension yields

$$\frac{1}{\eta}\{\hat{l}^{np}_{s,i} - \hat{l}^{p}_{s,i}\} = \hat{Y}^{np}_{s} - \hat{Y}^{p}_{s} - \frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}^{np}_{s,i}\bar{l}^{np}_{i}(\boldsymbol{s})dP(\boldsymbol{s}) + \\ + \frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}^{p}_{s,i}\bar{l}^{p}_{i}(\boldsymbol{s})dP\,(\boldsymbol{s}) - \hat{c}^{np}_{s,i} + \hat{c}^{p}_{s,i}. \tag{C15}$$

With (B8), (C15) becomes

$$\frac{1}{\eta}\{\hat{l}^{np}_{s,i} - \hat{l}^{p}_{s,i}\} = \alpha\left[\left\{\frac{1}{\bar{K}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{np}_{s,i}\,\bar{k}^{np}_{i}(\boldsymbol{s})dP(\boldsymbol{s}) - \frac{1}{\bar{K}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A}\hat{k}^{p}_{s,i}\bar{k}^{p}_{i}(\boldsymbol{s})\right\} - \\ - \left\{\frac{1}{\bar{L}^{np}}\sum_{i=1}^{I}\int_{E\times K\times A} e\,\hat{l}^{np}_{s,i}\bar{l}^{np}_{i}(\boldsymbol{s})dP(\boldsymbol{s})\,dP(\boldsymbol{s}) - \frac{1}{\bar{L}^{p}}\sum_{i=1}^{I}\int_{E\times K\times A} e\hat{l}^{p}_{s,i}\bar{l}^{p}_{i}(\boldsymbol{s})\,dP(\boldsymbol{s})\right\}\right] + \\ + (-1)\left[\hat{c}^{np}_{s,i} - \hat{c}^{p}_{s,i}\right]. \tag{C16}$$

The first bracket of the right hand side of (C16) is strictly negative due to (C8) and $\alpha > 0$. Moreover, the second bracket of the right hand side of (C16) is also strictly negative due to (C9). As the right hand side of (C16) is strictly negative, the left hand side of (C16) must be strictly negative as well. Thus, as $\frac{1}{\eta} > 0$, for any given *i*,

$$\hat{l}^{np}_{s,i} - \hat{l}^{p}_{s,i} < 0. \tag{C17}$$

[step 5] Whether an increase in individual labor supply volatility due to PAYG public pension, which is shown in (C17) of the above step 4, decreases volatility of social welfare or not is shown by examining the sign of $-\frac{ds\widehat{w}_s}{d\hat{l}_{s,i}}$ for arbitrarily given *s* and *i*. Notice that

$$\frac{ds\widehat{w}_s}{d\hat{l}_{s,i}} = \frac{ds\widehat{w}_s}{d\hat{v}_{s,i}}\frac{d\hat{v}_{s,i}}{dl_{s,i}}\frac{dl_{s,i}}{d\hat{l}_{s,i}}. \tag{C18}$$

Since $\bar{l}_i > 0$, $u_l < 0$ and $\frac{d\hat{v}_{s,i}}{dl_{s,i}}\frac{dl_{s,i}}{d\hat{l}_{s,i}} = \frac{u_l \bar{l}_i}{[\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\boldsymbol{s})dP(\boldsymbol{s})]}$,

$$sign\left(\frac{d\hat{v}_{s,i}}{dl_{s,i}}\frac{dl_{s,i}}{d\hat{l}_{s,i}}\right) = -sign\left(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\boldsymbol{s})\, dP(\boldsymbol{s})\right). \tag{C19}$$

From (C13), (C18), and (C19),

$$sign\left(-\frac{ds\widehat{w}_s}{d\hat{l}_{s,i}}\right) = \tag{C20}$$

$$= sign\left(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\boldsymbol{s})\, dP(\boldsymbol{s})\right) sign\left(\sum_{i=1}^{I}\int_{E\times K\times A} V_i(\boldsymbol{s})\, dP(\boldsymbol{s})\right) > 0.$$

When aggregating changes in social welfare from differences in individual labor supply volatility which are caused by PAYG public pension, (C17) and (C20) imply that increased labor supply volatility due to PAYG public pension reduces volatility of social welfare.

[step 6] Since individual utility function depends only on consumption and labor supply of each individual, by the definition of social welfare (i.e., the sum of the utilities of all individuals), the above step 3 and step 5 imply that

$$s\widehat{w}_s^{np} - s\widehat{w}_s^{p} > 0. \tag{C21}$$

That is, PAYG public pension reduces volatilities of social welfare over the business cycle.

Notes

1. For example, among the OECD countries, the average debt of PAYG public pension is projected to reach 124.2% of GDP by 2030 (Disney, 2000). Moreover, in the United States, funds for Social Security retirement program (PAYG public pension of the US) are expected to be depleted by 2033. In fact, as Pestieau and Ponthiere (2012) pointed out, rapid demographic aging imposes challenges on various public policies including public pension system.

2. In the literature and policy debates on PAYG public pension reforms, privatization of PAYG refers to eliminating PAYG public pension so that post-retirement consumption of individuals is financed only by their own private savings (instead of letting private sector provide PAYG pension). On the other hand, as a milder remedy to fiscal unsustainability of PAYG public pension system than its privatization, in 2013, Spain reformed its public pension system by introducing "sustainability factor" to adjust public pension benefit (Solé *et al*., 2018).

3. For instance, according to the OECD dataset, in 2015, the average size of public pension spending of the OECD economies is 7.5% of GDP.

4. In fact, a small number of countries combined defined contribution pension plans with PAYG finance method, instead of defined benefit pension plan, so that the benefit may vary after retirement reflecting population aging, which is called "notional defined contribution" system. However, in light of the motivation of this paper (fiscal unsustainability problem of traditional PAYG public pension), we focus only on the traditional PAYG system that presets benefit formula.

5. Even if an age-i individual is not eligible to receive public pension benefit in period t because he is younger than public pension entitlement age (i.e., $1_R(i)=0$), the public pension contributions that he has made can determine the amount of "potential" public pension benefit $a_{t,i}$, according to the given public benefit formula.

6. Although Lugauer (2012) found that GDP volatility is correlated with the population share under age 35 with the US data, he used labor search model without allowing employed individuals to freely choose different hours of work, unlike the model of this paper.

7. Because public goods are non-rivalrous and non-excludable, they do not make a difference in decision rules of individuals and the firm. Nevertheless, the government budget is affected by expenses on public goods. Thus, the term G appears only in (11) and (13), in the same way taken by numerous previous studies including Nishiyama and Smetters (2007). Above all, the main findings of this paper do not depend on whether the term G is included or not.

8. It is a simplification that public pension benefit is automatically given to benefit-eligible individuals once their age is equal to or greater than the public pension entitlement age R. In practice, in some countries, individuals can receive a discounted public pension benefit before reaching the entitlement age R. However, such an early take-up of discounted public pension benefit does not change the theoretical findings of this paper but adds complexity.

9. As PAYG public pension links the current population even to *unborn* future generations, the consumption stability resulting from the intergenerational risk-sharing of PAYG public pension cannot be achieved without PAYG public pension.

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Resumen

Utilizando un modelo de generaciones solapadas en el que individuos heterogéneos determinan su consumo y su oferta de trabajo para responder a los *shocks* en la productividad total de los factores, se constata que el sistema de pensiones de reparto produce un *trade off* entre la estabilidad macroeconómica y la variación en los niveles de bienestar. Este documento demuestra desde un punto de vista teórico que un sistema de pensión pública basado en el reparto puede reducir la volatilidad del consumo y del bienestar social a costa del aumento en la volatilidad de la producción agregada, en la oferta de trabajo y en la inversión. Al reducir los efectos de las recesiones en el nivel de renta en la etapa de jubilación, un sistema de pensiones públicas de reparto puede hacer que los individuos trabajen y ahorren menos en las recesiones y más en la etapa expansiva del ciclo.

Palabras clave: pensión pública de reparto, estabilidad macroeconómica, volatilidad del bienestar social.

Clasificación JEL: H55, E62, E32.