



Fiscal Responsiveness to Public Debt: An Analysis of Regional Debt Limit Uncertainty in Spain*

JOAN MARIA MUSSONS OLIVELLA**

Generalitat de Catalunya and Universitat de Barcelona

Received: November, 2018

Accepted: June, 2019

Abstract

The present analysis addresses the primary balance response to the debt of Spanish Autonomous Communities (ACs) from 1987 to 2015. Overall, the results provide evidence of a nonlinear relationship between these variables, which points to a situation of fiscal fatigue and reduced fiscal space. In 2015 the regional debt ratio stood at 26.8% of Gross Value Added, slightly above our lower estimate for the debt limit (24%) and well below the upper estimate (37%). We obtain lower debt limit thresholds when controlling for the extraordinary liquidity support measures provided by the central government. A reduced fiscal space in ACs calls for measures to ensure a sustainable debt trajectory and regain some room to deal with potential negative shocks.

Keywords: Public debt, debt sustainability, debt limit, uncertainty, fiscal reaction function.

JEL Classification: E62, H62, H63, H74.

1. Introduction

Concerns over fiscal sustainability in advanced economies have increased in recent years as a result of the widespread deterioration of public finances. The effects of the international financial crisis, but also population ageing and the stagnation of potential GDP, are elements that have put pressure on the public finances of these economies.

As a result of this context, the institutional framework has been modified, especially in Europe amid the Eurozone's sovereign debt crisis. More specifically, the system of fiscal

* This paper is part of my PhD research directed by Núria Bosch and Josep Lluís Raymond, to whom I am extremely grateful. I am also grateful to two anonymous referees and the editors of this journal for excellent comments. I also appreciate comments from Valèria Molina, Julio López Laborda, António Afonso, Natàlia Mas, Raul Ramos, Javier Pérez, Montse Bassols, Marta Curto and seminar participants at the Generalitat of Catalonia (December, 2017) and the XXV *Encuentro de Economía Pública* (Valencia, January 2018).

** ORCID ID: 0000-0001-7191-6487.

governance in the euro zone has been changed, with the implementation of stricter fiscal rules and a greater emphasis on the sustainability of public debt. At the same time, there has been a remarkable fiscal consolidation process, amidst an unfavourable macroeconomic environment. In many countries, including Spain, one can identify a “fiscal fatigue” situation that starts in 2014 (AIREF, 2016) and leads towards a slightly expansive fiscal policy after four years of contraction.

The present analysis builds on the methodology of the fiscal reaction function (FRF), where the key element in ensuring debt sustainability is the response of the primary balance to the accumulation of debt (see the seminal contribution of Bohn, 1998). This author suggests that sustainability would be achieved if at least the primary balance could grow linearly with the debt-to-GDP ratio (especially for high levels of this ratio). Other recent works have explored the implications for debt sustainability of a nonlinear relationship between both variables (see e. g. Ghosh *et al.*, 2013). In fact, an increasing or at least linear response of the primary balance cannot be ensured for any level of debt-to-GDP ratio. In this context, the concepts of “fiscal fatigue”, “debt limit” and “fiscal space” take prominence, as we will see later on. “Fiscal fatigue” is indicative of situations where the response of the primary balance to the accumulation of debt would be to decrease, while “debt limit” indicates the threshold from which the dynamics of debt become explosive. Finally, “fiscal space” estimates the distance from the debt-to-GDP ratio to the above-mentioned debt limit.

The present analysis addresses the primary balance response to the debt of Spanish Autonomous Communities (ACs) from 1987 to 2015 and evaluates a potential nonlinear fitting between both variables. It also presents an empirical reflection on the concepts of fiscal space and debt limit for ACs as a whole, paying special attention to uncertainty issues. This robustness assessment of the debt limit is not common in the literature (one exception is Ghosh *et al.*, 2013), and seems appropriate when working with finite samples, especially when an asymmetric distribution lies behind the statistical relationship to be tested.

The focus on the intermediate level of government is relevant for several reasons. First, Spanish ACs are responsible primarily for health, education and social services, that is, an essential part of the welfare state services. Second, the debt increase of ACs has been very significant, going from 5.7% of GDP in 2007 to 24.4% in 2015 (see Table 1). That is, four times bigger, by contrast to the dynamics of the general government which also experienced a significant debt increase, but with less intensity. Finally, to the best of our knowledge, the debate on the so-called debt limit hasn’t been raised in the case of Spanish ACs as yet.

Overall, the results of this study provide evidence of a nonlinear relationship between the primary balance and the public debt of ACs, which points to a situation of fiscal fatigue. These results have a backward-looking orientation, that is, fiscal reaction for the coming years is not necessarily determined by past behaviour. In 2015 the debt ratio stood at 26.8% of Gross Value Added (GVA), slightly above our lower estimate for the debt limit (24%) and well below the upper limit estimate (37%). However, these estimates are lower when controlling for the extraordinary liquidity support measures provided by the central government. That being said, a reduced fiscal space in ACs calls for measures to ensure a sustain-

able debt trajectory and regain some room to deal with potential negative shocks (Ghosh *et al.*, 2013).

Table 1
DEBT ACCORDING TO THE EXCESSIVE DEFICIT PROCEDURE BY
SUBSECTORS OF GENERAL GOVERNMENT
As a percentage of GDP

	2007	2015	2018	2015 / 2007
TOTAL	35.6	99.3	97.2	2.8
Central government	29.5	87.0	86.7	2.9
Social Security funds	1.6	1.6	3.4	1.0
Autonomous Communities	5.7	24.4	24.3	4.3
Local governments	2.7	3.3	2.1	1.2

Source: Bank of Spain.

This analysis is divided into three sections. After the introduction, we present the analytical framework adopted to address the response of the primary balance to debt, based on Bohn's seminal work (1998) and the latest contributions from Ghosh *et al.* (2013). In this second section, we also explore the main features of the institutional framework which play a crucial role in the fiscal behaviour of Spanish ACs. In the third section, we present an empirical application of the debt limit for the case of ACs. First, we estimate a FRF and then we assess the regional debt level in a stochastic setting. The assessment of uncertainty provides a brief reflection on inference by confidence intervals (in Appendix 3). Finally, the analysis ends with a conclusion section.

2. Analytical and institutional framework

2.1. Analytical framework: the primary balance's response to debt, the debt limit and the fiscal space

Until Bohn's seminal contribution (1998), the econometric approach to the sustainability of public debt was based on unit roots analysis of the debt-to-GDP ratio. However, the consistency of this analysis depends on a sufficient sample size and a proper specification of the deterministic component. As a matter of fact, an incorrect specification of the deterministic component, e. g. stemming from the existence of structural changes, can lead to inconsistent results (Perron, 1989).

Bohn (1998), however, addresses the sustainability of public debt from a new point of view and focuses on the response of the primary balance to changes in the ratio of public debt-to-GDP. This empirical strategy evaluates, through a FRF, the corrective actions in budgetary terms after an increase in the amount of debt¹. If the response of the primary

balance to an increase in the debt-to-GDP ratio is positive, one could conclude that the debt ratio is stationary and that would ensure the intertemporal budget constraint. In other words, sustainability would be achieved if the primary balance grows at least linearly with the debt-to-GDP ratio (and especially for high levels of this ratio). Bohn's work (1998) provides evidence of such corrective action for the US in the period 1916-1995. We now present a simple version of a FRF:

$$pb_t = \mu_t + f(d_{t-1}) + \varepsilon_t \quad (1)$$

where pb_t is the primary balance (as % of GDP), μ_t captures all the systematic determinants of the primary balance other than the lagged debt (i.e. the economic cycle, the election cycle, the institutional framework...), d_{t-1} stands for the lag of public debt (as % of GDP) and ε_t is the error term (which usually follows a white noise process).

This methodology doesn't require assumptions about interest rates and GDP growth, which implies an advantage over other sustainability assessments based on the standard debt accumulation equation, as discussed below. However, it should be noted that the methodology of FRF is a backward-looking approximation to sustainability, since it analyses the past behaviour of fiscal policy².

Next, we present an extension of this strand of research (Abiad and Ostry, 2005), that combines the FRF framework with the standard debt accumulation equation, which includes an approach to the so-called "debt limit" and the concept of "fiscal space". Therefore, from the standard debt accumulation equation we have (for simplicity, stock-flow adjustments are not included):

$$D_t = (1 + i_t) D_{t-1} - PB_t \quad (2)$$

$$d_t = \frac{1 + r_t}{1 + g_t} d_{t-1} - pb_t \quad (3)$$

where D_t is public debt, PB_t refers to the primary balance, i the nominal implicit interest rate, r the real implicit interest rate and g the real growth rate of GDP. Lower case letters indicate that the variable is expressed as % of GDP.

The expression of equation (3) in forward-looking terms, under the assumption of a constant trajectory for the primary balance, the real implicit interest rate and the rate of economic growth, leads to expressions (4) and (5) that allow us to calculate the debt limit (DL) and the fiscal space (FS_t)³:

$$DL = \frac{pb^{FRF}}{r - g} \quad (4)$$

The adjusted value of the primary balance is obtained from a FRF (pb_i^{FRF}) while the fiscal space measures the distance of the current level of debt to the debt limit (as a percentage of GDP).

$$FS_t = DL - d_t \quad (5)$$

The work of Ghosh *et al.* (2013) is very similar to Abiad and Ostry (2005), although it increases the flexibility of the primary balance's response to debt by applying a polynomial of degree 3. This functional form allows the so-called fiscal fatigue behaviour to be incorporated, that is, a decreasing response of the primary balance from a certain debt threshold.

The approach to the debt limit in section 3 follows that of Ghosh *et al.* (2013), as it considers a nonlinear relationship between the primary balance and debt, although it also introduces some differences. More specifically, we focus our attention on the polynomial containing the response of the primary balance to the accumulated debt, without applying the FRF to the standard debt accumulation equation [as in (4)]. Thus, we avoid making assumptions on the differential between the implicit interest rate and the growth rate of GDP. In other words, our approach is an ex-post analysis of ACs' fiscal behaviour, by contrast to the normative analysis emerging from expression (4). Our analysis is not about how ACs should behave but how they have behaved. It is an approximation to the confidence region where debt accumulation becomes explosive. It is also worth taking into consideration that the normative analysis of Abiad and Ostry (2005) may make no sense in a low real interest rate environment, and in particular when the denominator of expression (4) takes negative values.

An essential point of this analysis is the uncertainty of these limits. In many of the studies disseminated by major international organizations, uncertainty is approached through a sensitivity analysis that considers different values for the differential ($r - g$). These studies often evaluate differentials from past sub-periods considered representative for the economy analysed. Overall, these limits are based on one-time estimates and therefore do not include information on their stochastic properties. An exception is the work of Ghosh *et al.* (2013), which analyses fiscal fatigue in a stochastic context. These authors provide the standard error of fiscal space estimates through the application of resampling techniques. Our analysis also provides a stochastic assessment of the debt limit by obtaining confidence intervals.

2.2. Institutional framework

Before moving to the empirical analysis, we discuss the main features of the decentralisation process in Spain. The territorial organisation was established by the 1978 Constitution, the Statutes of Autonomy, and the Law Regulating Local Finances. The devolution of powers was gradual. In the 1980s, education and health responsibilities were devolved to seven ACs (Andalusia, Canary Islands, Catalonia, the Valencian Community, Galicia, the Basque Country and Navarre), whereas in the rest of ACs the decentralization of such powers started in the late 1990s and was completed in the early 2000s. Further, the Constitution enabled two fiscal regimes with significant differences concerning fiscal autonomy and per capital resources. This generated a large fiscal asymmetry between the ACs in the Foral Regime (Basque Country and Navarre) and in the Common Regime (the rest of ACs).

Besides, it is essential to note the uneven distribution of public expenditure and revenue among tiers of government. In 2011, one-third of non-financial public expenditures was ACs' responsibility, whereas ACs were responsible for 23.4% of non-financial public revenues

(Mussons Olivella, 2017). Such partial decentralisation is generally connected with lower sub-central fiscal performance (Rodden, 2002).

Revenue decentralization is not the focus of this paper, but it is important to state that, since 2009, the Common Regime operates under a partial equalisation mechanism. This system has several shortcomings, most noticeably, the substantial and arbitrary differences in per capita resources across ACs. In 2015, per capita resources ranged from below 8% to 30% above the average (Generalitat of Catalonia, 2018).

In addition to the degree of fiscal decentralisation, fiscal rules and the debt relationship between tiers of government are also important determinants of ACs' fiscal responsiveness to public debt.

First, in the aftermath of the financial and sovereign debt crisis in Europe, additional fiscal rules were required within the framework of the Stability and Growth Pact, as a response to widening fiscal imbalances. In the Spanish case, the Law on Budgetary Stability (in force since 2012) establishes a set of fiscal discipline principles based on the principles of budgetary stability (implemented through a structural balance rule and an expenditure rule) and financial sustainability (implemented through a debt ceiling). These legal provisions envisage a common debt threshold of 13% of GDP for all ACs to be reached in 2020. Besides, there is a process to set deficit targets between tiers of government, but in practical terms, its distribution is a unilateral decision of the central government. All ACs have the same debt targets regardless of their stabilisation and sustainability needs.

Second, extraordinary liquidity support measures were put forward by the central government in the aftermath of the financial crisis, when regional access to financial markets was limited or non-existent. On the one side, the Fund for the Financing of Payments to Suppliers was created in March 2012 to pay off debt with suppliers. In other words, trade credits (included as total liabilities of general government debt) were replaced by Excessive Deficit Procedure (EDP) debt. On the other side, the Regional Government Liquidity Fund (FLA) was created in July 2012 to finance the repayment of outstanding regional government debt and its authorised net borrowing. The Fund for the Financing of Regional Government assumed both funds in December 2014. Overall, the percentage of regional government debt held by the central government stood at 47.7% in 2015 (Delgado *et al.*, 2017 b). This centralisation of public debt issuance ensured ACs financial sustainability but it involved greater fiscal conditionality. See Gordo *et al.* (2013) regarding debt developments in Spain since the start of the crisis.

3. An empirical exercise for Spanish Autonomous Communities

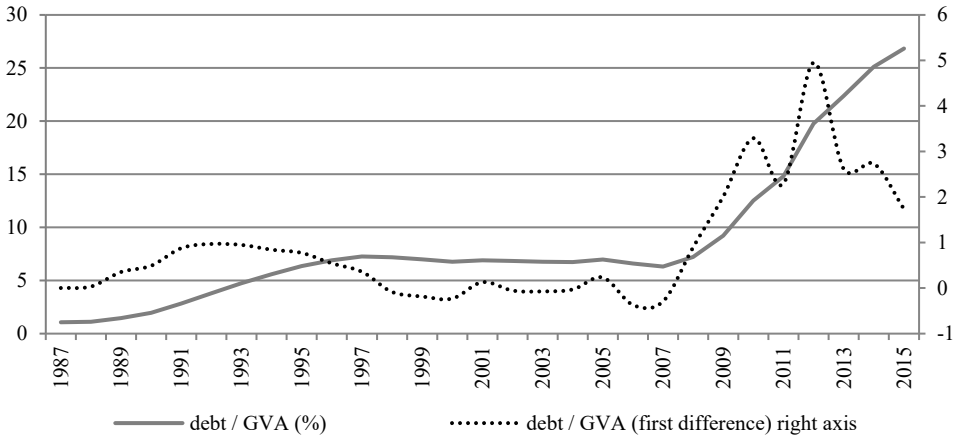
There is not much literature that has addressed the estimation of FRF at the regional level, and as far as we know, none of the previous works have considered a regional approach to the debt limit. In most of the research mentioned below for the Spanish case, the response of the primary balance to debt accumulation is linear or not significant, apart from the contribution of Esteller-Moré and Solé-Ollé (2004), who identify a nonlinear adjustment from a certain thresh-

old (and in particular under the budget consolidation scenarios in place between 1992-2001). However, that contribution focused on the financial bailout hypothesis and the period of analysis was distant from the current situation. Argimón and Hernández de Cos (2012) analysed the key factors of budget balance dynamics, that is, economic, political and institutional factors, for the period 1984-2004. In that case, the response of the Primary budget balance (PBB) to the debt ratio was supposed to be linear, although the results were not significant. Mussons Olivella (2017) has presented a similar analysis for the period 1987-2012, although the impact of the economic cycle and the institutional framework were dealt with in greater depth. That work identifies a corrective action (in linear terms) of the primary budget balance to the accumulation of debt, although from 2009 onwards the fiscal performance of ACs is weaker. Hernández de Cos and Pérez (2013), in their analysis of regional debt dynamics, also included the impact of the institutional framework, but the variables that capture market discipline are the key determinants. These authors also identify (for the period 1995-2010) a corrective action (in linear terms) to the accumulation of debt. Fourthly, Molina-Parra and Martínez (2015) introduced the role of vertical interactions between tiers of government, and specifically the impact of the central government's budget balance on regional finances. Their specification suggests a linear adjustment to the relationship between the primary balance and debt accumulation, although the effect is not significant. The period analysed in this case was 1995-2010. Leal Marcos and López Laborda (2015) also used the FRF's analytical framework, not to examine the determinants of the budget balance or the change in ACs' debt, but the deviations of budget balances from budgetary targets set by the central government. The debt specification is linear, although the effect is not significant. The period covered is shorter but closer in time, that is from 2003 to 2012. Lastly, the recent works of Lago Peñas *et al.* (2017) and Delgado-Téllez *et al.* (2017a) also analyse the determinants of fiscal non-compliance, but without including the debt ratio as an explanatory variable. Both papers make use of implicit interest rates as a proxy to market-financing costs. The former obtains no statistically significant impact on fiscal compliance, whereas the latter provides significant evidence of the positive impact of regional financing costs on compliance margins (that is, the difference between fiscal outturns and targets).

Before turning to the estimation of the FRF and fiscal space of ACs, we show (in Figure 1) the evolution of regional debt since 1987, the first year of the so-called final stage of the financing system. In the previous period (the pre-autonomic stage and the subsequent transitional stage), ACs' debt played a marginal role; in fact, in 1987, the debt ratio stood at 1.1% of GVA. In the period 1987-1997, we see a significant growth in debt, which can be attributed in part to the unfolding of the "autonomic system"⁴. The decentralisation of responsibilities was based on a cost-effectiveness criterion, where the agreed amounts were updated according to the dynamics of the central government's tax revenues. However, Lago Peñas (2002) and Mussons Olivella (2009) provide cross-sectional evidence of the higher debt levels of those ACs that were the first to assume the responsibilities of health and education. Both works also indicate that the less indebted ACs were those receiving more funds⁵. The analysis of Lago Peñas (2002) covers the period 1984-1996, whereas that of Mussons Olivella (2009) refers to 1996-2006. Throughout the period 1997-2007 regional debt stabilized around 7% of GVA, thanks to the expansion phase related to the real estate cycle. The end of this growth model leads to a period where debt gets into an explosive trajectory, moving from 6.3% in 2007 to 19.8% in 2012. In the last stage, the period 2012-2015, regional debt continues to

grow, but at a significantly slower pace thanks to the strengthening of fiscal consolidation. In 2015 debt reached 26.8% of GVA.

Figure 1
EVOLUTION OF ACS' DEBT (% of Gross Value Added)



Sources: Bank of Spain, BDMORES (Ministry of Economy and Finance) and INE.

3.1. Fiscal reaction function

The estimation of the FRF follows the empirical specification of Mussons Olivella (2017), which includes a detailed analysis of the impact of factors unrelated to the process of debt stabilization, especially those related to the economic cycle and the institutional framework. The empirical strategy involves a panel data analysis that reflects ACS' fiscal performance over the period 1987-2015. The estimation method deals with endogeneity problems by means of two-stage least squares. Our variable capturing the economic cycle is the variation in the unemployment rate and this is instrumented through the output gap of Spain's main trading partners, following a proposal by Galí and Perotti (2003)⁶. Our first stage results are reported in Table A1 (see Appendix 2). The estimates also take into account the contemporary correlation of the disturbance terms by applying the SUR methodology (Zellner, 1962). In this regard, we do not model central government behaviour, but we do control for common shocks to all ACs. For instance, we might expect that a central government measure that affects the primary budget balance in one AC would simultaneously affect the primary budget balance in other ACs as well. Note that these common shocks are captured by expression (7).

$$PBB_{it} = \gamma_1 + \gamma_2 PBB_{it-1} + f(DEBT_{it-1}, DEBT_{it-1}^2, DEBT_{it-1}^3) + \sum \gamma_j X_{jit+} + u_{it} \quad (6)$$

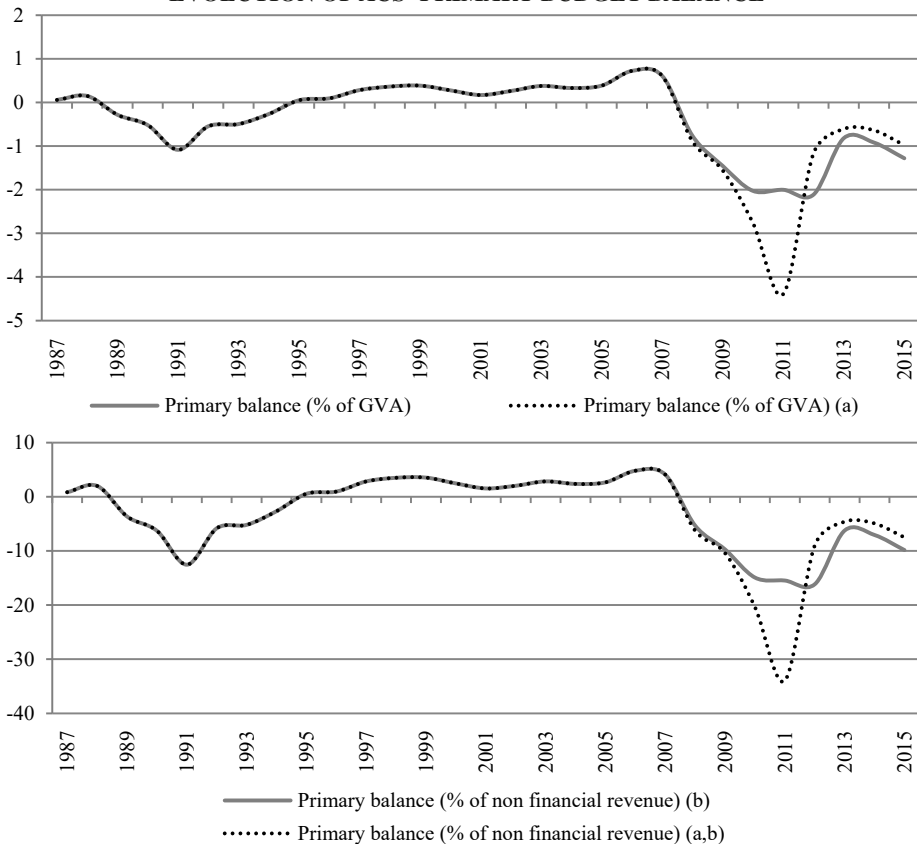
$$E(u_{it}, u_{kt}) = \sigma_{ik} \text{ for all } t, E(u_{it}, u_{ks}) = 0 \text{ for all } t \neq s \quad (7)$$

Our specification takes the *primary balance-to-GVA ratio*⁷ of each AC as the policy instrument and sets its target as a function of the *lagged dependent variable*, the *lagged debt*

ratio, and a vector of control variables X_j including cyclical conditions (these being captured by the change in the unemployment rate), an *index of expenditure responsibilities*⁸, an *electoral cycle variable* and variables capturing the annual amount of the *extraordinary liquidity support measures* as a percentage of GVA. In this last regard, we control for the regional government liquidity fund (FLA) and for all extraordinary liquidity support measures. See Appendix 1 for variable definitions and data sources.

We allow for a flexible response of the primary balance to the lagged debt ratio, by assessing a linear, quadratic and cubic specification. Besides, we also consider a different reaction across ACs to lagged debt, in connection with the proposal of Everaert and Jansen (2018). The index of expenditure responsibilities proxies the increase in regional per capita expenditure needs due to the assignment of health care and education (see Sorribas-Navarro, 2011), while the electoral cycle variable is a dummy for election years.

Figure 2
EVOLUTION OF ACS' PRIMARY BUDGET BALANCE



Notes: (a) Adjusted by 2008 and 2009 negative settlements and outstanding payments to suppliers. (b) Adjusted by FEOGA and local funding revenue.

Sources: Spanish Ministry of Finance and Civil Service and INE.

Before moving to the debt limit issue, it is worth commenting briefly on the FRF's empirical estimates (Table 2). The PBB is not sensitive to the cyclical conditions of the whole sample, but before 2008 [see specifications (5) and (6) in Table 2] the reaction to changes in the unemployment rate was negative, and thereby counter-cyclical as a whole (i.e. considering the impact of both the cyclical and discretionary component of fiscal policy)⁹. The results also point to a strong budgetary inertia, in accordance with the related literature. Our political and institutional variables also present the expected sign. The primary balance deteriorates in election years and as a result of the assumption of greater spending responsibilities. Last but not least, the response of the primary balance to lagged debt is non-significant in the whole sample, although the response previous to 2008 is positive, suggesting a corrective action to the accumulation of debt. We also provide a quadratic and cubic debt specification to enable more flexible responses. The quadratic specification [see specification (8)] is more robust, as the coefficient of the cubic term [see specification (9)] is only significant at the 10% level¹⁰. Therefore, the results point to a nonlinear response to be explored in the rest of the paper. A robustness check changing our dependent variable provides similar results (see Table A2 in Appendix 2). In that case, the primary budget balance to non-financial revenue is our dependent variable, following a proposal by Lago Peñas and Fernández Leiceaga (2013)¹¹.

Table 2
ACS' FISCAL REACTION FUNCTIONS: THE RESPONSE OF THE PRIMARY BALANCE TO DEBT

Dependent variable: Primary budget balance/Gross Value Added

	all ACs					
	(4)	(5)	(6)	(7)	(8)	(9)
Constant term (x1000)	0.68 (1.40)	0.32 (0.51)	-1.59** (-2.32)	-0.17 (-0.28)	-0.70 (-0.98)	-1.31 (-1.62)
Unemployment rate (first difference)	-0.09*** (-7.72)	0.01 (0.21)	-0.10*** (-3.42)	-0.08** (-7.12)	0.02 (0.53)	0.01 (0.43)
Unemployment rate (first difference) x dummy 08-15			0.42*** (9.18)			
Primary budget balance /Gross value added (-1)	0.48*** (-12.62)	0.52*** (-12.95)	0.46*** (14.94)	0.49*** (12.78)	0.53*** (13.61)	0.53*** (13.74)
Index of expenditure responsibilities (x 1000)	-0.89*** (-3.26)	-1.17*** (-3.52)	-1.40*** (-3.26)	-1.30*** (-3.53)	-1.47*** (-3.63)	-1.69*** (-4.1)
Electoral cycle (dummy) (x1000)	-2.53*** (-7.62)	-1.87*** (-4.64)	-1.90*** (-5.07)	-2.38*** (-6.49)	-1.79*** (-4.53)	-1.75*** (-4.42)
Debt/Gross value added (-1)	-0.01* (-1.79)	0.00 (0.07)	0.07*** (7.76)	0.03** (3.03)	0.03*** (3.66)	0.06*** (3.25)
Debt/Gross value added (-1) x dummy 2008-15			0.02* (1.86)			
(Debt / Gross value added (-1))²				-0.10*** (-4.61)	-0.10*** (-4.54)	-0.32** (-2.56)

(Continued.)

	all ACs					
	(4)	(5)	(6)	(7)	(8)	(9)
(Debt / Gross value added (-1))³						0.41* (1.83)
Dummy 2008-2015			-0.02*** (-12.54)			
Number of observations			493			
Sample			1987-2015			
Adjusted R2	0.42	0.34	0.73	0.41	0.36	0.37
Estimation method	OLS	IV	IV	OLS	IV	IV
Hausman exogeneity test						
t-statistic / F-statistic		-4.00	21.8		-4.06	-4.03
(p-value)		(0.00)	(0.00)		(0.00)	(0.00)

Notes: All regressions are estimated by Panel EGLS (Cross-section SUR weights). *** **, * and * statistically significant at 99%, 95% and 90%. t-statistics within brackets. Changes in the output gap of the 5 biggest Spanish export markets –weighted by their export shares– are used as an instrument of ACs' cyclical position. The primary balance is adjusted for 2008 and 2009 negative settlements, and outstanding payments to suppliers. Hausman exogeneity tests based on artificial regressions. First, we regress potentially endogenous variable on all exogenous variables and instruments and obtain the residuals. Second, we test for the significance of these residuals in the structural equation. Both regressions are done by applying OLS.

However, this nonlinear response may not be a general characteristic of the fiscal reaction function shared by all ACs in our panel, as suggested by Everaert and Jansen (2018) in a panel of OECD countries over the period 1970-2014. Thus, we allowed for heterogeneity in the reaction of the primary balance to lagged debt, and effectively, our findings point to significant differences across regions (Wald heterogeneity tests for equality of coefficients confirm this result, both in the linear and the quadratic case)¹². However, these regional estimates are not as robust as the average response for all the regions. In other words, there are significant differences across ACs, but we consider that our dataset is limited in size (and variability) to offer a robust fiscal reaction function for each region. ACs' high indebtedness is a relatively recent phenomenon, and thereby it may be premature to draw firm conclusions on region-specific FRF.

At this point, it is worth pointing out an important caveat for the debt limit study at the regional level, that is, the possibility of a bail-out. Debt limit estimates may be overstated in a situation where the central government raises funds on the market when regional access to financial markets is limited or non-existent. This situation was the general picture for ACs from 2012 onwards when extraordinary liquidity support measures were put forward. Thus, as a robustness check, we control for the extraordinary liquidity support measures in our baseline quadratic specification (see Table A3 in Appendix 2). As we can appreciate, the quadratic debt term is more negative in specifications (14)-(15) than in (8), and thereby fiscal fatigue sets in before. However, our analysis is only considering one direction of causality, as

the total amount of such liquidity may be a function of the situation of ACs' public finances. Thus, this robustness analysis should be regarded as a first approximation to the primary balance response conditioned to the introduction of these extraordinary mechanisms. Besides, we also evaluate our estimates for a restricted sample (1987-2011), before the adoption of these measures [see specification (13)]. As will be discussed in the next section, this latter specification also provides lower debt limit estimates than specification (8).

3.2. Analysis of the regional debt limit uncertainty

This section analyses the implications of the primary balance's nonlinear response to the ACs' lagged debt. To take account of estimation uncertainty we provide confidence intervals based on the estimator's empirical distribution. The approximation to the regional debt limit is made through simulations of the primary balance's response to the accumulation of debt on the basis of the quadratic specification of the FRF [see specification (8) in Table 2]:

$$\begin{aligned} \widehat{PBB}_{it} = & \widehat{\gamma}_1 + \widehat{\gamma}_2 \Delta UR_{it} + \widehat{\gamma}_3 PBB_{it-1} + \widehat{\gamma}_4 IRES_{it} + \widehat{\gamma}_5 ECYCLE_{it} \\ & + \widehat{\gamma}_6 DEBT_{it-1} + \widehat{\gamma}_7 DEBT_{it-1}^2 \end{aligned} \quad (8)$$

Ghosh *et al.* (2013) and Berti *et al.* (2016) take into account all the explanatory variables in obtaining the specific limit that makes debt unsustainable. However, other studies, such as that of Bohn (1998), pay attention exclusively to the primary balance's response to debt. In fact, the impact of budgetary inertia on our FRF plays a major role in the adjusted value of the primary balance (as the coefficient for the lagged dependent variable is 0.53). Therefore, for the purpose of calculating the debt limit one may exclude this inertia from the deficit (given the size of the budgetary imbalances in the aftermath of the crisis) and only analyse the primary balance's response to debt. This is our option [see below our equation (9)], and thereby these estimates are informative about ACs' public debt sustainability risks in a balanced budget setting.

$$\widehat{PBB}_{it} = \widehat{\beta}_1 DEBT_{it-1} + \widehat{\beta}_2 DEBT_{it-1}^2, \text{ where } \widehat{\beta}_1 = \frac{\widehat{\gamma}_6}{(1-\widehat{\gamma}_3)} \text{ and } \widehat{\beta}_2 = \frac{\widehat{\gamma}_7}{(1-\widehat{\gamma}_3)} \quad (9)$$

The inference by confidence intervals can stem from either the estimator's asymptotic distribution or the empirical distribution. However, it is shown in Appendix 3 that the asymptotic distribution of the estimator may lead to a biased inference in finite samples, particularly when an asymmetric distribution lies behind the statistical relationship to be tested (e. g. the nonlinear relationship considered in our paper). In such cases it is best to proceed from an empirical distribution obtained through resampling techniques.

Thus, in order to capture the uncertainty of the response of the primary balance to debt accumulation, we apply simulation techniques to obtain 1,000 vectors (Y) with the same distribution as $\widehat{\beta}$. Accordingly, we define a random variable $\tilde{\beta}$ centred at $\widehat{\beta}^0$, whose covariance matrix is $\widehat{cov}(\widehat{\beta})^0$ [as we do not observe $cov(\widehat{\beta})$] and where upper index 0 indicates a realization of the data generating process¹³.

$$\tilde{\beta} \sim N \langle \hat{\beta}^0, \widehat{cov}(\hat{\beta})^0 \rangle \quad (10)$$

$$Y = PZ + \hat{\beta}^0, \quad (11)$$

where $\widehat{cov}(\hat{\beta})^0 = PP'$ and $Z = P^{-1}(\tilde{\beta} - \hat{\beta}^0) \sim N \langle 0, I \rangle$.

At this point, it can be shown that Y presents the same distribution as $\tilde{\beta}$:

$$E(Y) = E(PZ) + E(\hat{\beta}^0) = E(\hat{\beta}^0) = \hat{\beta}^0 \quad (12)$$

$$COV(Y) = E(Y - \mu)(Y - \mu)' = E(PZZ'P') = PE(ZZ')P' = PP' = \widehat{cov}(\hat{\beta})^0 \quad (13)$$

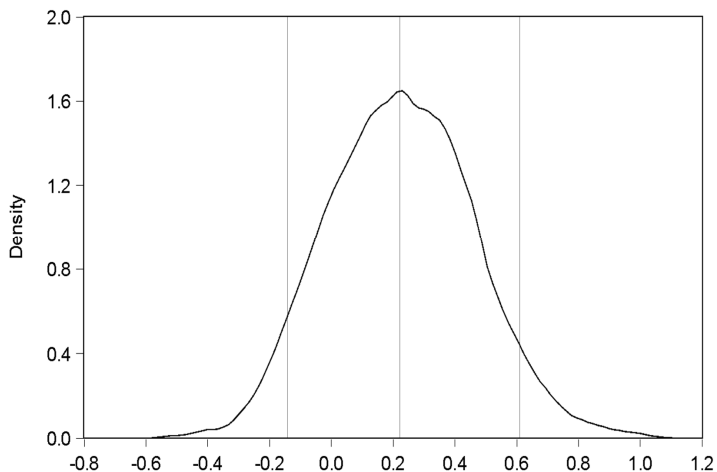
Therefore,

$$Y \sim N \langle \hat{\beta}^0, \widehat{cov}(\hat{\beta})^0 \rangle \quad (14)$$

The results of the simulations performed are presented below. Figure 3 shows the density function of the simulations for the adjusted value of the primary balance. This function includes 1,000 responses to a debt ratio of 26.8%, which was the latest available data for all ACs at the time of performing this analysis (i. e. 2015). As for the midpoint of the confidence interval we assign the median simulation, as the density function is almost symmetrical¹⁴. Figure 4 displays the reaction of the primary balance for a realistic range of debt ratios (from 0 to 45% of GVA). The adjusted value of the primary balance, as shown, has a parabolic shape with positive values up to the debt limit. The 90% confidence interval of this debt limit is (24%, 31%, 37%), where these values refer to the lower, middle and upper limit respectively, putting the debt ratio in 2015 slightly higher than the lower limit. Consequently, in 2015, the fiscal space for all ACs is 0% for the lower limit simulation, 4% for the mid-point simulation and 9% for the upper limit. Another interesting finding is that related to the “debt region” (i. e. the interval) where fiscal fatigue sets in, which is the region where the primary balance response follows a decreasing pattern. In this case, the 90% confidence interval of this fiscal fatigue threshold is (12%, 15%, 18%).

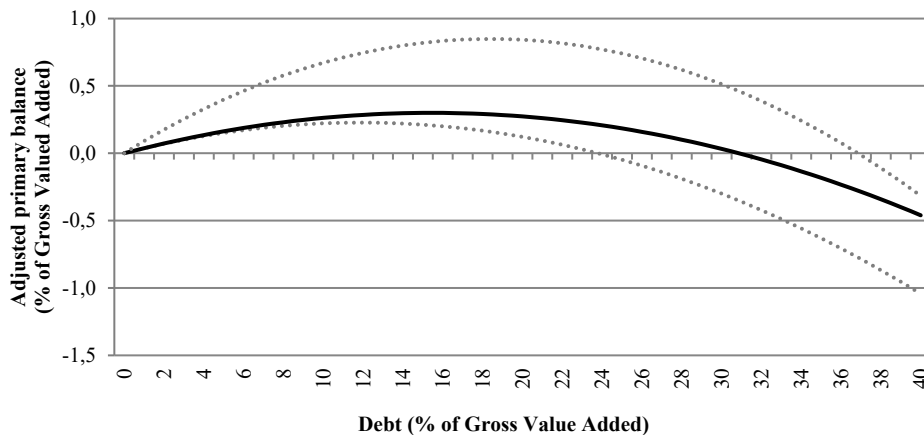
These results should be placed in context. First, one should not draw normative conclusions for specific ACs, although they call for measures to ensure both a sustainable debt trajectory and appropriate fiscal responsiveness regarding plausible adverse shocks (Ghosh *et al.*, 2013). This analysis contributes to the discussion regarding the debt sustainability of all ACs as a whole but does not attempt to carry out simulations for specific ACs. However, we do find significant differences across regions in the response of the primary balance to lagged debt and accordingly differences in terms of their debt limits. Besides, we should take into account that these thresholds not only vary across ACs but over time. In other words, past debt trajectories have an incidence on estimated fiscal space, especially when these trajectories have covered a limited debt range. This latter caveat refers primarily to the difficulties in assessing region-specific debt limits. Thus we consider that it would be better not to infer region-specific recommendations, but a general one. In particular, this heterogeneous response may suggest considering different debt targets, which differ from the current legal provisions for ACs (that envisage a common debt threshold of 13% of GDP). However, we should improve the measurement of this heterogeneous behaviour to offer more detailed proposals.

Figure 3
DENSITY FUNCTION OF THE RESPONSE OF THE PRIMARY
BALANCE TO THE 2015 ACS' DEBT RATIO



Note: The vertical bars indicate the midpoint and ends of the 90% confidence interval. This density function is based on 1,000 simulations drawn from specification (8) of Table 2. The simulations refer to the 2015 ACS' debt ratio.

Figure 4
RESPONSE OF THE PRIMARY BALANCE OF ACS TO DEBT: A SIMULATION WITH A
QUADRATIC FUNCTIONAL SPECIFICATION



Notes: Simulations for a 90% confidence interval based on specification (8) of Table 2, where only primary balance to debt reaction is taken into account (i. e. we disregard the effects of the rest of the explanatory variables)

Second, international evidence on fiscal space estimates should be taken into account, taking advantage of the contributions of Ghosh *et al.* (2013) and Nerlich and Reuter (2015). The former provides evidence for 23 advanced economies over the 1970-2013 period, whereas the latter deals with EU27 countries over the 1985-2013 period. In general, Ghosh *et al.*

(2013) suggest a decline in the response of primary balance to lagged debt as the debt ratio approaches 90-100% of GDP. This response would become negative around 150% of GDP. Estimates for the Spanish general government are close to this figure, as the projected debt limit stands at 153.9%. These results are similar to those of Nerlich and Reuter (2015).

Third, other approaches provide interesting insights for the Spanish case (not regarding ACs but the general government). In this regard it is worth mentioning the works of Andrés *et al.* (2017) and Domenech and González-Páramo (2017). The former explores the probability that the public debt exceeds a given threshold in connection with a market risk assessment. More specifically, this analysis builds on the VAR methodology to determine a threshold that maximizes the correlation between a measure of public debt risk and the sovereign spread. This measure of public debt risk is approached by the probability that public debt exceeds a given threshold over a certain time horizon. Thus, these results cannot be directly related to the notion of debt limit but to the region where risks to fiscal sustainability increase. Their findings suggest there has been a “prudent debt level” within the range of 50%-55% of GDP since 1999. The paper by Domenech and González-Páramo (2017) is based on the seminal contribution of Blanchard (1984), which takes into account the standard debt accumulation equation and an FRF to capture the convergence of the primary balance with its maximum level. This analysis does not provide a debt limit, but a combination of debt and primary balance positions ensuring fiscal sustainability. Estimates suggest that debt sustainability was “on a knife-edge” from 2009 to 2011 (in other words, Spain had no fiscal space during those years). It is also worth mentioning their evidence concerning high public debt outcomes. Their estimates point out that for each 10 percentage point increase in public debt, GDP falls 0.8%. In addition, their work also provides an interesting discussion on strategies to regain fiscal space. Their findings put structural reforms in place as an essential measure in restoring a prudent debt level. In this regard, a decrease of the structural unemployment rate in Spain from 15.5% to 7.5% would lead to a 6 percentage point improvement in the structural primary balance.

Fourth, we must also bring in the available evidence suggesting that central governments are shifting the burden of consolidation towards lower tiers of government (Von Hagen and Foremny, 2013; Vammalle and Hulbert, 2013; Foremny *et al.*, 2017). This evidence indicates that transfers from CG are cut during consolidation periods, especially when the fiscal autonomy of SCG is low. The Generalitat of Catalonia (2017) has provided evidence for the Spanish case on the deficit adjustment distribution over the period 2010 to 2016. ACs accounted for 46.5% of general government adjustment, local governments 23.5% and CG & Social Security 30%. This situation may have further contributed to the deterioration of welfare state services provided by ACs. However, the central government had to cope with the combination of a sharp increase in unemployment benefits during the crisis and the rigidity of social benefits expenditure (specifically old age pensions). Nevertheless, it is also true that available fiscal capacity is much more significant in the case of the central government, in particular concerning indirect taxation.

Fifth, we should examine our results concerning Spanish budgetary stability regulations, in addition to the proposal mentioned above about different debt targets. The 2012 Budget

Stability Law establishes a regional debt ceiling of 13% of GDP to be reached in 2020. According to our estimates, the above-mentioned threshold is close to the lower limit of our fiscal fatigue region, which suggests that the current ACs debt threshold is placed in a prudent area. A separate issue is the (in)feasibility of the debt target on the 2020 horizon. In fact, according to AIREF (2017) the fulfilment of this target is not expected before 2030 for most ACs.

Sixth, strictly speaking, our approach (based on Bohn, 1998, and Ghosh *et al.*, 2013) is designed for general governments, namely for sovereign states. In our setting, we should take into account from 2012 onwards the intervention of the central government to raise funds on the market when regional access to financial markets is limited or non-existent. This intervention may have avoided the default of some ACs, as it is likely that some of them had been exceeding their debt limit previously. Thus, our debt limit estimates may be overstated, as suggested when taking into account extraordinary liquidity support measures (see Table A4 in the Appendix 2)¹⁵.

4. Conclusions

The primary balance's response to the accumulation of debt is the central question addressed in this analysis, with two notable contributions: evidence is provided on the state of fiscal fatigue of Spanish ACs using the FRF methodology as well as on the uncertainty of the thresholds that indicate fiscal fatigue and debt unsustainability. To the best of our knowledge, we are not aware of a similar approach in the Spanish regional context. In addition, the stochastic approach of these thresholds is not a common practice in the literature. As has been noted, this uncertainty assessment is particularly valuable when the FRF's parameters present robustness problems. In our case, these issues are caused by a nonlinear and finite sample setting.

The results suggest a limited fiscal space for ACs as a whole and are thus indicative of the relative need for fiscal consolidation (inversely proportional to the estimates of fiscal space). A prudent fiscal position advises a reduction of the debt-to-GVA ratio, in order to ensure a sustainable debt trajectory and to regain the capacity to respond to potential negative shocks (Ghosh *et al.*, 2013). In this regard, it seems essential to (i) lay out and enforce reliable and feasible consolidation plans, (ii) enforce structural reforms to boost potential output (Domenech and González-Páramo, 2017), (iii) reform the regional financing system in order to ensure an appropriate distribution of resources that guarantees a sustainable balance between the principles of autonomy and equity, while leaving behind the preservation of the *statu quo*, and (iv) ensure vertical equity between tiers of government over time, that is a fair distribution of resources in connection with changes in expenditure needs (see a proposal by Castells *et al.* 2004). At this point, it is worth mentioning that the success of fiscal consolidation programs is not only about enforcing fiscal rules, but also about creating an equitable distribution of the burden of fiscal consolidation (Delgado-Téllez *et al.*, 2017a) and striking the right balance between stabilisation and sustainability needs¹⁶. In this regard, findings concerning a heterogeneous response to lagged debt suggest considering different debt targets across ACs.

Moreover, the stringency of fiscal rules should be balanced against fiscal autonomy, bearing in mind the Spanish decentralised constitutional structure (Foremny, 2014).

The debt limit debate is especially active in international bodies such as the IMF (Ghosh *et al.*, 2013), the European Commission (Berti *et al.*, 2016), and even in rating agencies (Zandi *et al.*, 2011), which perform frequent assessments of the debt limit for the countries they monitor. The present analysis aims to bring the regional perspective to a country level debate. In any case, this analysis should be weighed with other approaches including the assessment of short-term risks and that of the standard debt accumulation equation. The latter would take into account the sustainability impact of the debt structure and the expectations around financial conditions and growth prospects¹⁷.

Appendix 1. Definition of variables and data sources

Debt: debt-to-GVA ratio according to Excessive Deficit Procedure. *Source:* Bank of Spain.

Electoral cycle: = 1 in election years. *Source:* own elaboration from the Spanish Ministry for Home Affairs. <http://www.infoelectoral.mir.es/>.

Extraordinary liquidity support measures: annual amount of the extraordinary liquidity support measures provided by the central government, as a percentage of GVA. *Source:* Spanish Ministry of Finance and Civil Service and INE.

Gross value added: *Source:* BDMORES (Spanish Ministry of Finance and Civil Service) and INE.

Index of expenditure responsibilities: index of relative expenditure decentralisation that proxies the increase in regional per capita spending needs due to the assignment of health care and education. It is measured in relation to the average per capita provision of public goods and services (other than health and education services). *Source:* Sorribas-Navarro (2011).

Primary budget balance: primary balance-to-GVA ratio. The primary balance stands for the government net borrowing or net lending excluding interest payments. This data is adjusted for 2008 and 2009 negative settlements and outstanding payments to suppliers. *Source:* Budget Settlement of ACs and Autonomous Cities. Spanish Ministry of Finance and Civil Service.

Regional government liquidity fund (FLA): annual amount of the regional government liquidity fund as a percentage of GVA. *Source:* Spanish Ministry of Finance and Civil Service and INE.

Unemployment rate: unemployed as a percentage of the labour force. *Source:* INE.

Appendix 2. First stage results and robustness checks

Table A1
FIRST STAGE RESULTS: INSTRUMENTING OUR CYCLICAL POSITION
VARIABLE

Dependent variable: Unemployment rate (first difference)

	all ACs		
	(1)	(2)	(3)
Constant term (x1000)	0.30 (0.22)	-0.43 (-0.29)	-1.37 (-0.76)
Primary budget balance /Gross value added (-1)	-0.59*** (-12.12)	-0.59*** (-11.8)	-0.59*** (-11.83)
Index of expenditure responsibilities (x 1000)	4.51*** (3.7)	3.99*** (3.04)	3.70*** (2.75)
Electoral cycle (dummy) (x1000)	-5.57*** (-3.74)	-5.49*** (-3.68)	-5.51*** (-3.69)
Debt/Gross value added (-1)	-0.07*** (-5.68)	-0.04 (-1.14)	13.78 (0.23)
(Debt / Gross value added (-1))²		-0.10 (-1.08)	-0.51 (-1.18)
(Debt / Gross value added (-1))³			0.79 (0.97)
Output gap EU5 (first difference)	-0.99*** (-18.36)	-0.99*** (-18.35)	-0.99*** (-18.18)
Number of observations	493	493	493
Sample	1987-2015	1987-2015	1987-2015
Adjusted R2	0.57	0.57	0.57
Estimation method	OLS	OLS	OLS

Notes: ***, ** and * statistically significant at 99%, 95% and 90%. t-statistics within brackets. The primary budget balance is adjusted for 2008 and 2009 negative settlements, and outstanding payments to suppliers.

Table A2
ACS' FISCAL REACTION FUNCTIONS: THE RESPONSE OF THE PRIMARY
BALANCE TO DEBT. A ROBUSTNESS CHECK

Dependent variable: Primary budget balance/Non-financial revenue

	all ACs		
	(10)	(11)	(12)
Constant term (x1000)	1.51	-22.05**	-22.50**
	(0.16)	(-2.22)	(-2.08)
Unemployment rate	0.16	0.05	0.05
(first difference)	(0.29)	(0.08)	(0.09)
Primary budget balance	0.53	0.52***	0.52***
/Non financial revenue (-1)	(14.25)***	(13.82)	(13.88)
Index of expenditure responsibilities	-7.89	-3.30	-3.27
(x 1000)	(-2.11)**	(-0.82)	(-0.81)
Electoral cycle (dummy) (x1000)	-14.31	-13.63***	-13.54***
	(-3.49)***	(-3.46)	(-3.49)
Debt/Non financial revenues (-1)	0.01	0.05***	0.05***
	(1.4)	(-5.79)	(2.75)
(Debt/Non financial revenues (-1))²		-0.02***	-0.02
		(-6.48)	(-1.08)
(Debt/Non financial revenues (-1))³			0.00
			(-0.01)
Number of observations		493	
Sample		1987-2015	
Adjusted R2	0.33	0.39	0.39
Estimation method		IV	
Hausman exogeneity test	-3.43	-3.47	-3.47
t-statistic (p-value)	(0.00)	(0.00)	(0.00)

Notes: See Table 2.

Table A3
ACS' FISCAL REACTION FUNCTIONS: CONTROLLING FOR THE EXTRAORDINARY LIQUIDITY SUPPORT MEASURES

Dependent variable: Primary budget balance/Gross Value Added

	all ACs			
	(8)	(13)	(14)	(15)
Constant term (x1000)	-0.70 (-0.98)	-3.83 (-0.28)	-0.85 (-1.21)	-0.71 (-1.03)
Unemployment rate (first difference)	0.02 (0.53)	-0.04*** (-7.12)	0.01 (0.45)	0.01 (0.28)
Primary budget balance /Gross value added (-1)	0.53*** (13.61)	0.64*** (12.78)	0.56*** (13.99)	0.56*** (13.57)
Index of expenditure responsibilities (x 1000)	-1.47*** (-3.63)	-2.11*** (-3.53)	-1.31*** (-3.39)	-0.96** (-2.16)
Electoral cycle (dummy) (x1000)	-1.79*** (-4.53)	-1.26*** (-6.49)	-1.39*** (-3.30)	-1.66*** (-3.73)
Debt/Gross value added (-1)	0.03*** (3.66)	0.19*** (3.03)	0.03*** (3.45)	0.03*** (2.71)
(Debt / Gross value added (-1))²	-0.10*** (-4.54)	-1.30*** (-4.61)	-0.17*** (-4.14)	-0.18*** (-5.00)
Regional government liquidity fund (FLA)			0.14*** (3.23)	
Extraordinary liquidity support measures				0.14*** (3.72)
Number of observations	493	425	493	493
Sample	1987-2015	1987-2011	1987-2015	1987-2015
Adjusted R2	0.36	0.59	0.36	0.42
Estimation method	IV	IV	IV	IV
Hausman exogeneity test t-statistic / F-statistic (p-value)	-4.06 (0.00)	-3.68 (0.00)	-3.90 (0.00)	-3.58 (0.00)

Notes: See Table 2.

Table A4
SUMMARY OF DEBT LIMIT ESTIMATES BASED ON QUADRATIC SPECIFICATIONS: CONTROLLING FOR THE EXTRAORDINARY LIQUIDITY SUPPORT MEASURES

Specification	Sample	Maximum response of primary balance (as a % of GVA)	Debt limit (as a % of GVA)	Control for extraordinary liquidity support measures
(8)	1987-2015	(11.9, 15.4, 18.4)	(23.8, 30.9, 36.8)	No
(13)	1987-2011	(6.7, 7.2, 7.7)	(13.4, 14.4, 15.4)	Non-implemented
(14)	1987-2015	(7.5, 10.2, 12.7)	(14.9, 20.4, 25.5)	Only FLA
(15)	1987-2015	(3.9, 7.4, 10.1)	(7.7, 14.8, 20.2)	All

Note: In brackets, you can find the midpoint and ends of the 90% confidence interval.

Inference by confidence intervals can be based on either the estimator's asymptotic distribution or the empirical distribution. It is shown below that the asymptotic distribution of the estimator may lead to a biased inference in finite samples, particularly when an asymmetric distribution lies behind the statistical relationship to be tested (e. g. the nonlinear relationship considered in our paper). In such cases it is best to proceed from an empirical distribution obtained through resampling techniques.

If we wish to obtain a confidence interval on the basis of the estimator's asymptotic distribution one could proceed from:

$$\hat{\theta} = f(\hat{\beta}) \approx f(\beta) + \frac{\partial f}{\partial \beta} (\hat{\beta} - \beta) \rightarrow N \left[f(\beta), \frac{\partial f}{\partial \beta} \text{cov}(\hat{\beta}) \frac{\partial f}{\partial \beta} \right], \quad (15)$$

where θ is any nonlinear combination of the β coefficients.

If we assume (as an example) that $f(\beta) = \frac{1}{\beta}$, it is easy to check that:

$$\hat{\theta} \rightarrow N \left(\frac{1}{\beta}, \frac{\sigma^2}{\beta^4} \right)$$

Let's consider two simple examples to illustrate the differences in the inference of confidence intervals:

Example 1.— $\beta \sim N(2,1)$

On the one hand, on the basis of the asymptotic distribution we obtain the 95% confidence interval, where the lower and upper bounds for *theta* are:

$$\hat{\theta} = 0.50$$

$$\hat{\sigma}_{\hat{\theta}} = 0.25$$

$$\text{Upper limit} = 0.50 + 0.50 = 1.00$$

$$\text{Lower limit} = 0.25 - 0.50 = 0.00$$

On the other hand, we may proceed from the empirical distribution:

First, we generate a sample of 100,000 observations from $\beta \sim N(2,1)$

Second, we estimate 100,000 *thetas* from $\hat{\theta} = \frac{1}{\hat{\beta}}$

Third, the estimated *thetas* are ordered from the smallest to the largest. The lower bound is obtained as the *theta* value corresponding to observation 2,501 and the upper bound as the *theta* value corresponding to observation 97,500. The results obtained through simulation are:

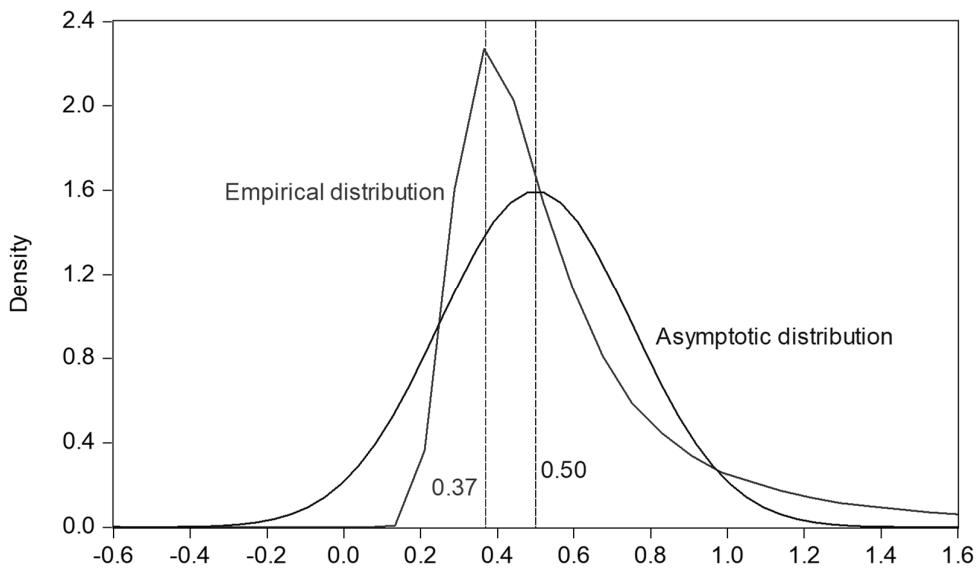
Upper limit = 3.00

Lower limit = 0.19

$\hat{\theta}$ mode = 0.37

Figure A1 displays the asymptotic and empirical distribution functions of theta. In this regard, recall that the latter has been obtained through simulation techniques.

Figure A1
EMPIRICAL AND ASYMPTOTIC DISTRIBUTIONS OF “THETA” ($\hat{\theta}$), EXAMPLE 1



Please note that the expected value of “theta” does not exist because it includes an expression that is the product of infinite and zero. That is to say:

$$E(\hat{\theta}) = \int \frac{1}{\hat{\beta}} p(\hat{\beta}) d\hat{\beta} = \infty * 0 = \textit{indeterminate} \tag{16}$$

Therefore, we include the mode as being our preferred estimator (as it is the most likely value), instead of the expected value of “theta”.

Example 2.— $\beta \sim N(1,1)$

If we proceed as in the first example, the results (see Figure A2) highlight even more clearly the inadequacy of using the asymptotic distribution as a basis for inference. On the one hand, on the basis of the asymptotic distribution, we obtain the following statistics:

$$\hat{\theta} = 1.00$$

$$\hat{\sigma}_{\hat{\theta}} = 1.00$$

$$\text{Upper limit} = 3.0$$

$$\text{Lower limit} = -1.0$$

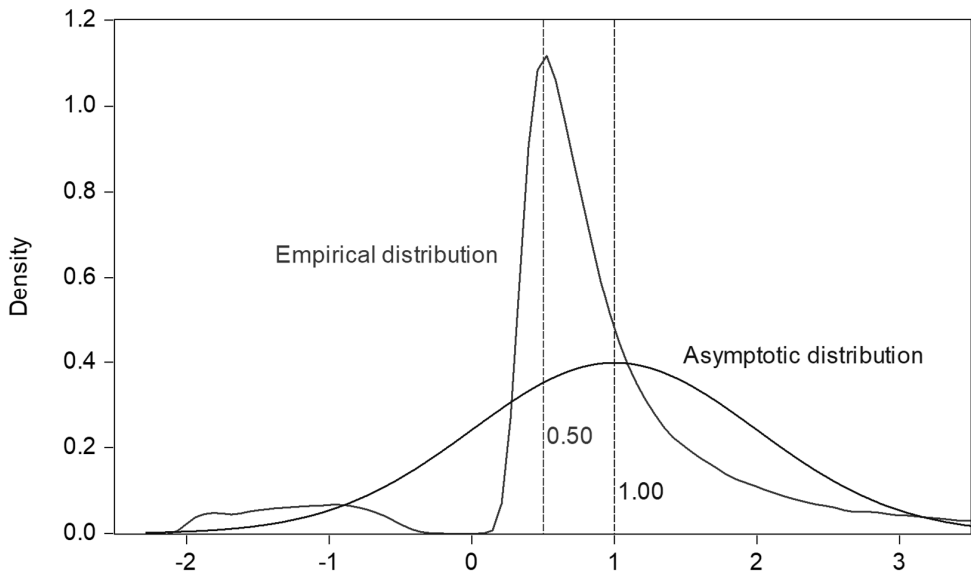
On the other hand, the empirical distribution provides the following statistics:

$$\text{Upper limit} = 10.07$$

$$\text{Lower limit} = -9.09$$

$$\hat{\theta} \text{ mode} = 0.50$$

Figure A2
EMPIRICAL AND ASYMPTOTIC DISTRIBUTIONS OF “THETA” ($\hat{\theta}$), EXAMPLE 2



Thus, both examples illustrate the benefits of operating from an empirical distribution when we are interested in building confidence intervals in a nonlinear and finite sample setting.

Notes

1. The research field based on these FRFs is very wide and, according to Checherita-Westphal and Zdarek (2017), covers three research areas: (i) one addresses the contrasts of the sustainability of public debt; (ii) the estimation of fiscal policy rules based on the literature on monetary policy rules and, lastly (iii) the estimation of FRFs as an input for simulations within the Debt Sustainability Analysis of major international organizations (DSA) or as input for the calculation of the so-called debt limits. For a summary of the literature on FRFs see the work of these authors, Checherita-Westphal and Zdarek (2017), and that of Berti *et al.* (2016).
2. The European Commission uses these FRF to evaluate the credibility of the stability and convergence programmes of Member States (see Berti *et al.*, 2016). It assesses to what extent the fiscal position projected in these plans (namely, the changes in the structural primary balance) differs from the position arising from these FRFs.
3. See Abiad and Ostry (2005) for a demonstration of expression (4).
4. This unfolding led to an asymmetric and gradual process of decentralisation of powers, where health and education stand out from the rest. Mussons Olivella (2009, 5) takes into consideration the years of transfer of these two major responsibilities, with substantial differences across ACs depending on the time they were devolved. In 2002, all ACs had assumed the responsibilities of health and education.
5. Lago Peñas (2002) takes into account the development funds, whereas Mussons Olivella (2009) considers both the development funds and revenue from the regional financing system.
6. Our choice to proxy the cyclical position was not an easy job. We tried different standard approaches (HP filter as well as unobserved components models), but we had robustness problems, particularly at the end of the sample. Therefore we turned to Gross Value Added (GVA), unemployment rate and employment data. In our point of view, GVA and unemployment are the best proxies to cyclical conditions, each indicator covering different information. On the one hand, GVA is the most comprehensive measure of economic activity, but it is not a “gap” variable. On the other hand, the unemployment rate is closer to a cyclical position variable, but it provides partial information (as it only takes into account labour market information). Fortunately, symmetric FRF are robust to both proxies. See Mussons Olivella (2017) for more details on these robustness results.
7. The primary budget balance of each AC is computed according to budgetary criteria both regarding the institutional scope as well as the accounting rules. This data relates to all the public entities included in the consolidated budget of each AC. In addition, we adjust the dependent variable according to De la Fuente (2013) and Lago Peñas and Fernández Leiceaga (2013). On the one hand, we make corrections for the negative settlements from the funding system (for years 2008 and 2009), the payments for which were deferred. We regard these negative results as cancelled in 2010 and 2011 respectively (as provided for initially by law), and accordingly we do not apply revenue withholdings from then on (to return those deferred payments). On the other hand, we make adjustments related to outstanding payments to suppliers, which are captured by annual changes in accounts payable for accrued liabilities. After these adjustments our data are very close to national accounts criteria. Figure 2 displays the dynamics of our dependent variable, as well as the relevance of the above-mentioned corrections.
8. The index of expenditure responsibilities (IER) is defined following Sorribas (2011, 169):

$$IER_{it} = (H_{it}EH + ED_{it}EE) / CE$$
 where H_{it} and ED_{it} are dummy variables that are equal to one if the region i has been assigned the provision of health or education, respectively, in period t ; $EH = 564.67 \text{ €}$; $EE = 428.05 \text{ €}$; $CE = 714.48 \text{ €}$ is the average per capita expenditure, at 2001 constant prices, during the period 1986–2001 on health, education, and on the provision of the public goods and services that are assigned to all regions, respectively. Hence, IER_{it} is equal to 0 if the region has not been assigned health neither education; is equal to 0.8 (0.6) if it is responsible for providing health (education); and, is equal to 1.4 if it is responsible for providing health and education.”
9. See Mussons Olivella (2017) for an analysis of ACs’ discretionary fiscal policy. This paper indicates that ACs’ fiscal stance has been pro-cyclical over the period 1987–2012. As a matter of fact, this pro-cyclicality has sharpened since the last crisis. This paper also provides more details on the role of institutional settings.
10. The specification of the debt polynomial is an empirical issue, as has been reflected in the literature. For example, we can see that the seminal work of Bohn (1998) considers a quadratic form while the pioneering contribution

from Ghosh *et al.* (2013) identifies a cubic specification. A review of the literature can be found in Checherita-Westphal and Zdarek (2015) and Berti *et al.* (2016), including the details of the econometric specifications.

11. We performed an additional robustness check regarding the inclusion of year fixed effects. Despite being significant from a statistical point of view, results did not bring important changes in the primary balance response to debt (and accordingly in the estimates of regional debt limit). In fact, changes in the unemployment rate, electoral cycle and our proxy to expenditure decentralisation capture most of the common dynamics across ACs.
12. These results are available on request.
13. Please note that $cov(\hat{\beta}) = \sigma^2(X'X)^{-1}$ in contrast to $\widehat{cov}(\hat{\beta}) = \widehat{\sigma}^2(X'X)^{-1}$.
14. If the density function displayed an asymmetric distribution, it could also be appropriate to use the statistical mode as the midpoint of the interval.
15. Our estimates for a restricted sample (1987-2011), before the adoption of these measures, suggest lower debt limit thresholds as well as lower uncertainty. In this case, the 90% confidence interval of this debt limit is (13%, 14%, 15%). We have also checked the sensitivity of our results for the whole sample (1987-2015) utilizing control variables. If we take into account the total amount of the regional government liquidity fund (FLA), debt limit estimates decreases around ten percentage points (15%, 20%, 25%) while if we consider the total amount of extraordinary liquidity support measures, these confidence region decreases five points more (8%, 15%, 20%). Overall, these latter results reinforce the necessity to regain fiscal space.
16. See a careful discussion in European Commission (2016b).
17. In this regard it is worth noting the DSA framework of the EC in European Commission (2016a) and that of the ECB in Bouabdallah *et al.* (2017). AIREF (2017) provides an empirical implementation of this framework for ACs.

References

- Abiad, A. and Ostry, J. (2005), "Primary Surpluses and Sustainable debt levels in emerging market countries", *IMF Policy discussion paper* 06.
- AIREF (2016), *Los retos fiscales de España: orientación a medio plazo y refuerzo institucional de la política presupuestaria*, presentation at Colegio de Economistas de Madrid, 15th of September.
- AIREF (2017), *Informe sobre las líneas fundamentales de los presupuestos para 2018 de las Comunidades Autónomas*, 24th of October.
- Andrés, J., Pérez, J. J. and Rojas, J. A. (2017), "Implicit debt thresholds: an empirical exercise for the case of Spain", *Bank of Spain, Documentos de Trabajo* No. 1701.
- Argimón, I. and Hernández de Cos, P. (2012), "Fiscal Rules and Federalism as Determinants of Budget Performance: An Empirical Investigation for the Spanish Case", *Public Finance Review*, 40 (1): 30-65.
- Berti, K., Colesnic, E., Desponts, C., Pamies, S. and Sail, E. (2016), "Fiscal Reaction Functions for European Union Countries", *European Commission Discussion Paper* 028, April.
- Blanchard, O. (1984), "Current and Anticipated Deficits, Interest Rates and Economic Activity", *European Economic Review*, 25(1): 7-27.
- Bouabdallah O., Checherita-Westphal, C., Warmedinger, T., de Stefani, R., Drudi, F., Setzer, R. and Westphal, A. (2017), "Debt sustainability analysis for euro area sovereigns: a methodological framework", *ECB Occasional Paper Series* No. 185, April.
- Bohn, H. (1998), "The Behaviour of U.S. Public Debt and Deficits", *The Quarterly Journal of Economics*, MIT Press, vol. 113(3): 949-963.

- Castells, A., Sorribas, P. and Vilalta, M. (2004), *Les subvencions d'anivellament en el finançament de les comunitats autònomes*, Barcelona, Institut d'Estudis Autònoms, Generalitat de Catalunya.
- Checherita-Westphal, C. and Ždarek, V. (2017), "Fiscal reaction function and fiscal fatigue in the euro area", *ECB Working Paper Series* No. 2036, March.
- De la Fuente, Á. (2013), "Las finanzas autonómicas en boom y en crisis (2003-12)", *Hacienda Pública Española* 205(2): 127-150.
- Delgado-Téllez, M., Lledó, V. D. and Pérez, J. J. (2017 a), "On the Determinants of Fiscal Non-Compliance: An Empirical Analysis of Spain's Regions", *IMF Working Paper* 17/5, January.
- Delgado, M., García, B. and Zubimendi, L. (2017 b), "Developments in public debt in Spain in 2016", *Economic Bulletin* 2/2017, Banco de España.
- Domenech, R. and González-Páramo, J. M. (2017), "Budgetary stability and structural reforms in Spain: lessons from the recession and options for the future", *BBVA Research, WP* 17/05.
- Esteller Moré, A. and Solé Ollé, A. (2004), "Estabilidad presupuestaria y financiación autonómica", *Hacienda Pública Española*, monographic issue: 173-201.
- European Commission (2016 a), "Fiscal Sustainability Report 2015", *European Economy Institutional Paper* 018, January.
- European Commission (2016 b), "Report on Public Finances in EMU 2016", *European Economy Institutional Paper* 045, December.
- Everaert, G. and Jansen, S. (2018), "On the estimation of panel fiscal reaction functions: Heterogeneity or fiscal fatigue?", *Economic Modelling*, 70: 87-96.
- Foremny, D. (2014), "Sub-national deficits in European countries: The impact of fiscal rules and tax autonomy", *European Journal of Political Economy*, 34, 86-110.
- Foremny, D., Sacchi, A. and Salotti, S. (2017), "Decentralization and the duration of fiscal consolidation: Shifting the burden across layers of government", *Public Choice*, 171(3): 359-387.
- Galí, J. and Perotti, R. (2003), "Fiscal policy and monetary integration in Europe", *Economic Policy*, 18: 533-572.
- Generalitat of Catalonia (2017), *The autonomous communities accounted for half of Spain's deficit adjustment between 2010 and 2016, The Catalan Economy*, Issue 8, July.
- Generalitat of Catalonia (2018), *Annual Financial Report 2017*.
- Hernández de Cos, P. and Pérez, J. J. (2013), "Sub-national public debt in Spain: political economy issues and the role of fiscal rules and decentralization", in *Fiscal relations across government levels in times of crisis – making compatible fiscal decentralization and budgetary discipline*, *European Economy, Economic Papers* 501, July.
- Gordo, L., Hernández de Cos, P. and Pérez, J. J. (2013), "Developments in Spanish public debt since the start of the crisis", *Economic Bulletin*, July-August, Banco de España: 19-36.
- Ghosh, A. R., Kim, J. I. Mendoza, E. G. Ostry, J. D. and Qureshi, M. S. (2013), "Fiscal fatigue, fiscal space and debt sustainability in advanced economies", *Economic Journal*, 123 (566): 4-30.
- Lago Peñas, S. (2002), "Nivelación interterritorial e ineficiencia de las decisiones presupuestarias: reflexiones para el caso español", *Hacienda Pública Española*, 162-3: 79-97.

- Lago Peñas, S. and Fernández Leiceaga, X. (2013), “Las finanzas autonómicas: Expansión y crisis, 2002-2012”, *Papeles de Economía Española*, 138: 129-146.
- Lago Peñas, S., Fernández Leiceaga, X. and Vaquero, A. (2017), “¿Por qué incumplen fiscalmente las Comunidades Autónomas?”, *Investigaciones Regionales*, 37: 7-29.
- Leal Marcos, A. and López Laborda, J. (2015), “Un estudio de los factores determinantes de las desviaciones presupuestarias de las Comunidades Autónomas en el periodo 2003-2012”, *Investigaciones Regionales*, 31: 35-58.
- Molina-Parra, A. and Martínez, D. (2015), “Do federal deficits motivate regional fiscal (im)balances? Evidence for the Spanish case”, *Documentos de Trabajo Centro de Estudios Andaluces*, DT 001/15.
- Mussons Olivella, J. M. (2009), “Diferències en el nivell d’endeutament de les CA: descentralització de competències i suficiència de recursos”, *Papers de Treball* Número 6/2009, Departament d’Economia i Finances, Generalitat de Catalunya.
- Mussons Olivella, J. M. (2017), “Fiscal Decentralization and the Cycle in Spain: an Empirical Analysis of Fiscal Policy Responses”, *Política Económica/Journal of Economic Policy*, vol. XXXIII(3), December: 305-344.
- Nerlich, C. and Reuter, W. H. (2015), “Fiscal rules, fiscal space and procyclical fiscal policy”, *ECB Working Paper* 1872.
- Perron, P. (1989), “The great crash, the oil price shock, and the unit root hypothesis”, *Econometrica*, 57: 1361-1401.
- Rodden, J. (2002), “The Dilemma of Fiscal Federalism: Grants and Fiscal Performance around the World”, *American Journal of Political Science*, 46(3): 670-687.
- Sorribas-Navarro, P. (2011), “Bailouts in a fiscal federal system: Evidence from Spain”, *European Journal of Political Economy*, 27: 154-170.
- Vammalle, C. and Hulbert, C. (2013), “Sub-national finances and fiscal consolidation: Walking on thin ice”, *OECD Regional Development Working Paper* No. 02, OECD Publishing.
- Von Hagen, J. and Foremny, D. (2013), “Fiscal federalism in times of crisis: Sharing the cost of adjustment”, in N. Bosch and A. Solé-Ollé (eds.), *IEB’s Report on Fiscal Federalism ’12*, Barcelona, Institut d’Economia de Barcelona.
- Zandi, M., Packard, T. and Cheng, X. (2011), “Fiscal Space: A New Gauge of Sovereign Risk”, *Moody’s Analytics*.
- Zellner, A. (1962), “An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests of Aggregation Bias”, *Journal of the American Statistical Association*, 57: 348-368.

Resumen

El presente análisis aborda la respuesta del saldo primario a la acumulación de deuda de las Comunidades Autónomas en el periodo 1987-2015. En conjunto, los resultados aportan evidencia sobre una relación no lineal entre dichas variables que apunta a una situación de fatiga fiscal así como a un espacio fiscal reducido. En el año 2015 la ratio regional de deuda estaba en el 26,8% del Valor Añadido Bruto, ligeramente por encima de nuestra estimación para el umbral inferior del límite de la deuda (24%) pero a una distancia notable del umbral superior (37%). Sin embargo, se obtienen umbrales más

pequeños cuando se controla por las medidas extraordinarias de liquidez aportadas por el Gobierno central. Un espacio fiscal reducido en el ámbito autonómico sugiere adoptar medidas para garantizar una trayectoria sostenible de la deuda y recuperar margen de maniobra para afrontar eventuales *shocks* negativos.

Palabras clave: deuda pública, sostenibilidad de la deuda, límite de la deuda, incertidumbre, función de reacción fiscal.

Clasificación JEL: E62, H62, H63, H74.

