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Indirect taxation, tax evasion and profits

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Abstract: In a duopoly with differentiated products under price competition, this paper analyses the firms' tax compliance behaviour in the presence of tax evasion to challenge the conventional wisdom that indirect taxes penalise profits. In contrast to the preceding literature, it is shown that indirect taxes can increase firms' profits. The appearance of this unconventional finding is more likely when there exists a sufficient competitive pressure in the industry and the effort for detecting evasion is not too strong.

JEL classification: H25, H26, H32

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1. Introduction

A well-established belief in the public finance literature is that indirect (either ad-valorem or specific) taxes in a standard imperfectly competitive market reduce firms' profits.[1] Another established fact is indirect tax evasion by the firms', a widespread phenomenon in many countries.

In fact, only restricting our considerations to the sales tax, it is sufficient to say that the revenues from taxes on general consumption (mainly the VAT) is about 18.9% of total tax revenues in countries belonging to the OECD (OECD, 2008) and 27.8% in 2018 for European Union countries (European Commission, 2020a).

VAT evasion is a concern (Keen and Smith, 2006), whose magnitude is remarkable: the European Commission (2020b) reveals that the VAT gap in 2017 and 2018 – mainly due to tax evasion - as a percentage of the VAT total tax liability is roughly 10% for the median country; however, for countries such as Greece and Romania is above 30%, and around 25% for Italy and Lithuania (see Figure 1).

Though the literature on tax evasion has pointed out the need for re-examining many results previously established by the tax analysis in the absence of evasion, the relationship between

[1] Nonetheless, this is not a universally valid insight. Katz and Rosen (1985), for instance, have shown that profits in oligopoly can increase with a higher tax rate, although in the absence of tax evasion activities. However, it should be noted that their demonstration is rather special because based on a numerical example of a duopoly with i) quadratic costs; ii) consistent conjectural variations with the corresponding parameter about *–0.61*; iii) taxation burdening only on the input factor prices. In fact, we show in the present paper that with Bertrand competition, linear costs and sales taxes in the absence of tax evasion profits always reduce with increasing tax rates.

[Figure 1 about here]

profits and taxes under evasion has not been so far fully explored. This is precisely the goal of this paper.

Virmani (1989) and Cremer and Gahvari (1999) study the impact of tax evasion in a perfectly competitive market whereas Marrelli and Martina (1988), Bayer and Cowell (2009, 2016), Besfamille at al. (2009, 2013), Goerke and Runkel (2006, 2011), Goerke (2017) and Fanti and Buccella (2021) analyse markets with imperfect competition.

A first set of those works, close to the present contribution, has analysed the impact of competition on tax evasion (e.g. Marrelli and Martina, 1988; Goerke and Runkel, 2006, 2011; Goerke, 2017; Fanti and Buccella, 2021). Marrelli and Martina (1988) assume that firms decide the evasion (in terms of unpaid taxes); however, they will incur a fine (including the tax) given by the evaded tax multiplied for a penalty rate (larger than one). Those authors show that more competitive markets lead to lower amounts of tax evasion, both in symmetric duopolies and asymmetric ones in which differences in production costs are not excessive. On the other hand, Goerke and Runkel (2011) assume that firms decide the evasion in terms of undeclared sales, and the imposed fine is an increasing, convex function in evaded revenues. They find that there is no clear-cut relationship between market competition and tax evasion: positive (negative) if demand is inelastic (elastic).

In a Cournot oligopoly with homogenous goods, Goerke (2017) investigates whether tax evading activities on operating profits affects the number of firms in a market, the excessive entry prediction, and the impact on social welfare.

Following the approach of Goerke and Runkel (2011), Fanti and Buccella (2021) further analyse the link between market competition and firms' tax evasion both in a quantity (Cournot) and price (Bertrand) duopoly with differentiated goods. Those authors show that a negative or a positive relation depends on the source of the competitive pressure (i.e. a marginal cost increase, higher product substitutability or a change in the competition mode) and the pre-existing level of competition.

A second group of works has analysed the role of tax enforcement of indirect taxation (e.g. Besfamille at al., 2009, 2013). Besfamille et al. (2009) study the relation between tax enforcement, output and the efficiency of the government in collecting tax revenues when firms in imperfectly competitive markets evade a specific output tax. On the other hand, in an economy in which firms compete on quantities and decide the fraction of sales taxes to evade, Besfamille et al. (2013) focus on the determinants of the enforcement level of indirect taxation in a positive setting.

A third set of papers has investigated the effects of tax evasion on efficiency and optimal commodity taxation (e.g. Virmani, 1989; Cremer and Gahvari, 1993, 1999). Virmani (1989) analyses indirect tax evasion in a competitive market and shows that production inefficiency leads to no tax compliance, and evasion can generate Laffer-type curves. Cremer and Gahvari (1993) investigate firms' tax evasion activities in a competitive economy in which the tax administration can detected illegal behavior through auditing, while Cremer and Gahvari (1999) study how tax evasion affects the link between marginal changes in tax rate, tax revenue, and social welfare.

Finally, scholars have analysed the impact of the audit rules by tax authorities on tax evasion. For instance, in a context of strategically interdependent firms, Bayer and Cowell (2009) analyse the impact of different audit rules on revenues from a tax on profits because the enforcement policy impacts both market decisions and compliance behavior. Bayer and Cowell (2016) further propose "smarter" audit policies that consider the link between a firm's reported profits and industry reports: this will create an externality for the decision makers that could affect firms′ reporting policies as well as their market decisions.

Those works, however, disregard the analysis of how indirect taxation affects profits. We study how sales taxation affects profits in an imperfectly competitive (duopoly) context in which firms may evade a part of such taxes.

In contrast to the conventional wisdom which suggests that indirect taxes on sales or output volume reduce profits, we show that firms' profits may actually increase. In particular, this counter-intuitive result is obtained when there exists a sufficient competitive pressure in the industry and the effort for detecting evasion is not too strong.

Two interesting policy implications emerges from this finding: 1) firms could support tax policies that not only, ceteris paribus as regards the level of tax revenues, are based on a mix of high taxes and low audit probability, but also, provided that the latter is sufficiently low, on higher taxes; 2) since also public revenues are increasing with taxes for most situations in which profits are increasing as well, it follows that firms and government may agree on higher taxes.

The paper is organised as follows. Section 2 presents the model and the examines the equilibrium properties. In particular, we discuss the effects of increasing tax rates on the firms' profitability as well as the resulting policy implications. Section 3 presents some concluding remarks and outline future lines of research.

2. The model and the equilibrium analysis.

2.1 The model

Following Fanti and Buccella (2021), we consider a Bertrand duopoly game in which firms have to pay an ad valorem sales tax; however, part of this tax can be evaded. [2] The representative consumer's preferences are (e.g. Singh and Vives, 1984):

$$U(q_i, q_j) = a(q_i + q_j) - \frac{\left(q_i^2 + 2\gamma q_i q_j + q_j^2\right)}{2} \tag{1}$$

where q_i and q_j define firm *i* and *j*'s output (*i*, *j* = 1, 2, *i* ≠ *j*), respectively, $a > 0$, and $\gamma \in (0,1)$ is the degree of product differentiation, with goods assumed to be imperfect substitutes. Goods are homogeneous when γ equals 1, and firms compete in the same market. On the other hand, when γ equals 0, goods are independent and each firm acts as monopolist in its market. Hence, the higher γ is, the higher is the product market interaction. From (1), one gets the linear (inverse) product market demand for firm *i* :

$$p_i(q_i, q_j) = a - \gamma q_j - q_i . \tag{2}$$

From (2) and its counterpart for firm *j*, the direct product demand for the firm *i* is:

$$q_i(p_i, p_j) = \frac{a(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)} \tag{3}$$

[2] We have chosen an ad-valorem instead of a specific tax because the former is more commonly used in many countries. Needless to say, also the case of specific tax is worth to be studied and is left to the future research.

The government levies a sales tax, $t \in (0,1)$. Firm i's true tax base is given by the sales revenue $p_i q_i$. To evade taxes, firms can understate to the tax authority their revenues declaring, $d_i \in [0, p_i q_i]$, as its tax base. Thus, firm i's unreported revenues are $p_i q_i - d_i$. The firm i's tax bill is td_i. Tax evasion is detected with a probability $m \in (0,1]$. If detected, firm i has to pay taxes on the true revenues, $p_i q_i$, plus a penalty $P\left[p_i q_i - d_i\right]$, which depends on evaded revenues. Thus, the expected penalty is $mP\left[p_i q_i - d_i\right]$, which firms consider as a tax avoidance's cost measure.[3]

We assume that the detection probability, m , is a constant parameter; on the other hand, the penalty function, P, is convex and strictly increasing in evaded revenues. Thus, the expected penalty (or the cost of avoidance), mP, increases and is convex in evaded sales. In line with Goerke and Runkel (2011, p. 732) and Fanti and Buccella (2021), we assume a quadratic penalty function

$$P(.) = \frac{\left(p_i q_i - d_i\right)^2}{2} \tag{4}$$

The rationale for the choice of the penalty function in (4) is twofold. First, it ensures analytical tractability and guarantees that, in the feasible set of the parameters of the model, an interior solution exists.[4] Second, as Goerke and Runkel (2011, p. 716) remark, the

[3] Note that it may also be alternatively assumed that the penalty is a function of the evaded taxes instead of the undeclared revenues. However, the equilibrium results are qualitatively the same.

[4] One can remark that the literature tends to use linear tax penalties. For instance, in Cremer and Gahvari (1993) detected non-complying firms pay a fine proportional to the amount of

penalties in evaded revenues usually increase with the severity and extent of insufficient tax payments. This seems to support the assumption in our model that the penalty function is increasing in evaded revenues. In addition, several penalty schemes include prison sentences for serious tax evasion activities. Therefore, if the penalty scheme contemplates not only monetary but also non-monetary penalties, such prison sentences suggest that the penalty is convex.[5]

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the tax evaded. However, differently from our work, increasing concealing sales to the tax administration entails a convex cost for firms. Piolatto (2015, Appendix) studies the possibility of introducing itemized (optimal) deductions to incentivize consumers to declare their purchases. As in this work, in an extension to the basic model, that author considers the case of deterring evasion via auditing activities and the imposition of fines to the firm which acts illegally. The penalty scheme is a linear; however, the cost of auditing is convex. Immordino and Russo (2018) study a cooperative model of tax evasion in which a seller and a buyer bargain a price reduction in case of a cash payment (without a receipt) that allows tax evasion, with both bargaining parties incurring a convex cost for evading tax. Technically, to guarantee and internal solution to the maximization problem and obtain an optimal level of tax evasion (concavity of the profit/utility function), one ingredient of the model has to be characterized by convexity.

[5] A few countries empirically show existent penalties coherent with the features of the penalty function proposed in this work. Countries such as Denmark and Spain (and, concerning interests to pay for late tax payments, also Ireland) have their financial penalties increasing in the sum of the tax evaded (see OECD 2009, 2011, 2013). From a theoretical perspective, Hashimzade et al. (2010) propose (in a related framework) the penalty function $\Phi = \phi E^{\gamma}, \gamma > 0$, in which ϕ is a positive, constant scale parameter and γ a government's endogenous choice parameter. When $\gamma \geq 1$, the punishment function Φ is convex. The

Firm i's cost function is $c_i q_i$, where c_i is the constant marginal cost, assumed uniform across firms, i.e., $c_i = c_j = c$. Hence, firm i's expected net profits are

$$E[\pi_i] = m\left\{(1-t)pq_i - cq_i - \frac{(pq_i - d_i)^2}{2}\right\} + (1-m)\{pq_i - cq_i - td_i\} \tag{5}$$

where the first term in brackets in Eq. (5) is firm i's profits when tax evasion is detected, while the second term is profits when such an evasion is undetected. In the current setting, the firms' pricing and evasion decisions are taken separately. To see this fact in a net manner, equation (5) can be rewritten as follows[6]

$$E[\pi_i] = (1-t)pq_i - cq_i + (1-m)te - m\frac{e^2}{2} \tag{5.bis}$$

where $e = (pq_i - d_i)$ is the under-reported sales. Moreover, the evasion decisions of the two firms are independent of each other. By substituting (3) in (5), firm i's expected net profit is given by

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authors show that "If the objective of the government is to control fraud it therefore has to choose a convex penalty with $\gamma > 1$".

[6] We thank an anonymous referee for having suggested this reformulation of Eq. (5).

$$E[\pi_i] = m\left\{ [(1-t)p_i - c]\frac{a(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)} - \frac{\left(p_i \frac{a(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)} - d_i \right)^2}{2} \right\}$$

$$+(1-m)\left\{ (p_i - c)\frac{a(1-\gamma) - p_i + \gamma p_j}{(1-\gamma^2)} - td_i \right\} \tag{6}$$

2.2 The equilibrium analysis

Taking as given the rival firm's price, firm i maximizes π_i, choosing price, p_i, and declared revenues, d_i, simultaneously. From the first-order conditions for an interior solution (see Fanti and Buccella, 2021, Appendix) the equilibrium price, declared sales revenue and output are

$$p_i = p_j = p = \frac{a(1-t)(1-\gamma) + c}{(2-\gamma)(1-t)}. \tag{7}$$

$$d_i = d_j = d = t\left(\frac{1}{m} - 1\right) + \frac{\lfloor a^2(1-t)^2(\gamma-1) - c\gamma a(1-t) + c^2 \rfloor}{k} \tag{8}$$

$$q_i = q_j = q = \frac{(2-\gamma)(1-t)[a(1-t) - c]}{k} \tag{9}$$

where $k = (2-\gamma)^2(1-t)^2(1+\gamma)$, and with the standard non-negative condition on output[7]

$$a \geq \frac{c}{(1-t)}. \tag{10}$$

[7] An analytical inspection of (9) reveals that a tax increase (decrease) yields a less than proportional decrease (increase) in the firms' output.

Since it always holds that $pq > d$,[8] the condition guaranteeing an interior solution for *d,* that is $d \in (0, pq)$ does exist, and boils down to the following inequality (see Fanti and Buccella, 2021)[9]:

$$m > m^{\circ} = \frac{t(4+\gamma^3-3\gamma^2)(1-t)^2}{a^2(1-t)^2(1-\gamma)+c\gamma a(1-t)-c^2+t(1-t)^2(4+\gamma^3-3\gamma^2)} \tag{11}$$

This implies that equilibrium profits (i.e. $\pi_i = \pi_j = \pi$) are:

$$\pi = \frac{\left[\begin{array}{l} m^2t^2(1-t)(2-\gamma)^2(1+\gamma)+ \\ 2m\left[a^2(1-t)^2(1-\gamma)-2ca(1-t)(1-\gamma)+c^2(1-\gamma)-t^2(1-t)(1+\gamma)(2-\gamma)^2\right]+ \\ +t^2(2-\gamma)^2(1-t)(1+\gamma) \end{array}\right]}{2m(2-\gamma)^2(1-t)(1+\gamma)} \tag{12}$$

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[8] In fact, it is easy to show that $(pq-d) = \dfrac{t(1-m)}{m} > 0$. The latter result can be obtained taking the first-order condition with respect to e in equation (5.bis) which leads to the profit-maximizing prices and the respective quantities in the Bertrand-Nash equilibrium as in equations (7) and (9).

[9] Note that the inequality (11) might also be expressed in terms of *a,* although this would be algebraically less tractable. In such a case it can be shown that the satisfaction of condition (11) would require a value of *a* sufficiently high. Therefore, it follows that the "feasibility" of the model requires a sufficiently high market "size", which is the usual requirement in the oligopoly literature with linear demand. Needless to say, all the results of the paper are achieved when first order conditions of (6) hold.

2.3 Profit and tax analysis

In this sub-section we investigate the effects of the sales taxes on profits. Such effects are detailed in the following propositions and remarks.

The marginal effect of changes in t on the level of the equilibrium profit is given by the following derivative:

$$\frac{\partial \pi}{\partial t}=\frac{(1+m^2)tk+m\left\lfloor a^2(1-t)^2(\gamma-1)-\left\lfloor c^2(\gamma-1)+2k\right\rfloor\right\rfloor}{mk}. \tag{13}$$

Analytical inspection of (13) yields the following result.

Proposition 1. *The relationship between profit and tax rate is U-shaped.*

Proof: See the Appendix.

The economic intuition behind Proposition 1 is as follows. Using the envelope function theorem and equations (7)-(9), the derivative of the expected profit with respect to the tax rate in equilibrium can be written as

$$\frac{dE\left[\pi_i\right]}{dt}=-p_iq_i+[(1-t)p_i-c]\frac{\partial q_i}{\partial p_j}\frac{dp_j}{dt}+\frac{(1-m)^2}{m}=$$

$$=-p_iq_i+[(1-t)p_i-c]\frac{\gamma}{1-\gamma^2}\frac{c}{(2-\gamma)(1-t^2)}+\frac{(1-m)^2}{m}$$

In the above expression, the first two terms describe the effect of tax on profits when evasion is not possible. The first term is the direct effect of tax, and it is always negative. The second term, where p_j defines the competitor's price, is the indirect effect of tax which reflects the degree of substitutability between the products ($\frac{\partial q_i}{\partial p_j} = \frac{\gamma}{1-\gamma^2} \gtrless 0$) and the response of the competitor's price to tax ($\frac{dp_j}{dt} = \frac{c}{(2-\gamma)(1-t^2)} > 0$ because the impact of taxation is that of increasing effectively the marginal cost of production). This term is clearly positive when the two products are substitutes ($\gamma > 0$) and its value is larger the closer is the degree of substitution among the two products, i.e. the more intense is the competition between firms.

The possibility of tax evasion does not alter the first two terms. Using the first-order condition, the effect of tax on the expected profit can be further rewritten as

$$\frac{dE[\pi_i]}{dt} = -p_i q_i \left[1 - \frac{\gamma \frac{c}{1-t}}{(1+\gamma)a + \frac{c}{1-t}} \right] + \frac{(1-m)^2}{m} t$$

The expression in the square brackets is positive for $\gamma < 1$, and therefore, in the absence of evasion the effect of tax on profit is always negative. The second term is the effect of tax on profit when there is opportunity to evade, and it is positive and increasing in tax rate. As a consequence, with evasion, starting at $t = 0$, a marginal increase in the tax rate leads to a fall in the expected profit; as t increases, the positive effect of the last term counterbalances the negative effect of the first term. Also, for any given tax rate, $\frac{(1-m)^2}{m} t = 0$ at $m = 1$, and as $m \to 0$, $\frac{(1-m)^2}{m} t \to \infty$ while the first term remains finite. As a consequence $\frac{dE[\pi_i]}{dt} < 0$ for $m = 1$ and $\frac{dE[\pi_i]}{dt} > 0$ for $m \to 0$. By continuity, for a given tax rate t^* there is

$m^* = m(t^*) \in (0,1)$ such that $\frac{dE[\pi_i]}{dt} = 0$. For this value m^* of the probability of detection, $\frac{dE[\pi_i]}{dt} < 0$ at $t = 0$. Since the last term increases in t, it is possible that for $m = m^*$, $\frac{dE[\pi_i]}{dt}$ becomes positive for $t = t^*$. To sum up, for a given configuration of parameters it is possible to find the probability of detection such that the expected profits increase with tax rate for sufficiently high tax rates.

Finally, the larger is γ, the smaller is in absolute value the negative effect of tax on the expected profits (the first term). Hence, t^* is lower for larger values of γ. In other words, when the degree of competition becomes fiercer, the expected profits are increasing in tax rate over a larger range of tax rates.[10]

To gain further insights, as an example we may fix *m=0.5* in (13), without loss of generality, which leads to an easily interpretable analytical expression. Then, the following inequality holds:

$$\frac{\partial \pi}{\partial t} \gtrless 0 \Leftrightarrow \left[tk - 2\left[a^2(1-t)^2 - c^2\right](1-\gamma)\right] \gtrless 0 \tag{14}$$

Equations (7) and (9) show that both prices and quantities are under-shifted by the tax rate. Eq. (14) reveals that, in the relevant range of the sales tax, profits decrease (increase) if the

[10] We are extremely grateful to an anonymous referee for suggesting this general explanation of Proposition 1.

reducing impact of taxation on quantities dominates (is dominated by) the increasing effect on prices. Moreover, from inspection of (14), the following Proposition also holds:[11]

Proposition 2. *If m=0.5, then the higher γ and c and the lower a, the more likely profits are increased by a tax rate increase.*

Now we may qualify the economic conditions under which the unconventional result above illustrated is more likely. In particular, we investigate the role played by the degree of competition existing in the market. As known, the degree of competition is inversely related to the firms' market power which, in turn, can be measured by the Lerner index, that is, the difference between the (after-tax) output price and marginal production costs, relative to the (after-tax) output price (e.g. Martin, 2001). The Lerner index is given by

$$L = \frac{\left[(1-t)p-c\right]}{(1-t)p} = \frac{(1-\gamma)\left[a(1-t)-c\right]}{a(1-t)(1+\gamma)+c} \tag{15}$$

Then the following Lemma holds:

Lemma 1 *The higher γ, t and c and the lower a, the higher is the degree of competition.*

Proof: see Fanti and Buccella (2021, Appendix).

[11] Needless to say, Proposition 2 holds true for all the other values of m in the domain of feasibility of the model (i.e. $m \in\ ]m°,\ 1]$), although this can not be shown by simple inspection.

Since changes of *t, c* and *a* are not only pure competition effects, then, to investigate the relationship between competition and effects of higher taxation on profits, we focus on changes of γ, whose reductions (resp. increases) have, according to the "differentiation principle", an univocal pro-collusive (resp. pro-competitive) effect. Fanti and Buccella (2021) provides an economic intuition behind Remark 1. For instance, focusing only on the parameter γ , one can observe from (15) that the total effect of marginal cost variations on the Lerner index can be decomposed in two effects. On the one hand, an increase in the degree of product differentiation expands output and decreases the equilibrium price (see (7) and (9), respectively). On the other hand, an increase in γ has a direct negative effect on the Lerner index, L. With a linear demand, all those effects lead an increase in γ to reduce unambiguously L, therefore increasing competition.

As a consequence of Lemma 1 and Prop. 2, the following Remark holds:

Remark 1. *The higher the degree of market competition due to a lower product differentiation, the more likely the positive effects of higher tax rates on profits occurs.*

The feasibility of the model requires that the first order conditions of (6) hold. Such conditions could be violated for a sufficiently high tax rate; therefore, it has to be ascertained that, for the range of sufficiently high tax rate values for which profit is increasing with increasing tax rates, the FOCs conditions are satisfied.

This investigation is not analytically tractable in general, and we resort to numerical simulations to illustrate the content of Proposition 1 as well as its quantitative relevance. Only for illustrative purposes, we fix the following parametric set: $a=3$, $c=0.7$, $m=0.40$, for which in the range of tax rate values ($t \in (0, 0.63)$) - which includes the most realistic tax rates values – FOCs hold.

[Figure 2 about here]

Figure 2 shows that profits are strongly increasing with the sales tax when the product substitutability is very high – for instance, for $\gamma=0.99$ as well as for $\gamma=0.90$ profits are even larger with a maximal tax rate about 62% than without taxes - and in any case profits may be increasing with tax rates when the latter are sufficiently high, provided that also the product substitutability is still sufficiently high (for instance $\gamma=0.80$). Note that, as regards all the figures shown in the paper, declared sales and output are always positive and thus they are not displayed for economy of space.

2.4 Discussion and illustration of the main results.

The intuition behind the results above presented is the following. The effects of tax rates on profits work through two channels: 1) the strategic competition effects, that is how indirect taxation influences the price decisions (effect that, of course, would be absent under perfect competition); 2) the tax evasion effect, that is how indirect taxation influences the evasion decisions (for given audit and penalty rules policies).

On the one hand, the role of the indirect tax rate on price competition is univocal: taxation works for reducing the firm's market power as shown by Lemma 1. Moreover taxation reduces profits through the direct effect of the fiscal burden. On the other hand, the indirect tax rate affects tax evasion decisions (and thus the profits) through the opposite effects of the expected penalty and of the undeclared sales in the following way. First, we state the following Lemma:

Lemma 2. *In the absence of tax evasion, profits are always reduced by sales taxes. Proof: Since profits are given by* $\pi^{NE} = \frac{(1-\gamma)[a(1-t)-c]^2}{k}$ *, then it is easy to see that* $\frac{\partial \pi^{NE}}{\partial t} = -\frac{(1-\gamma)\lfloor a^2(1-t)^2 - c^2 \rfloor}{k} < 0$.

This shows that, under Bertrand competition, the conventional wisdom holds in the absence of evasion. For a better understanding of how this conventional wisdom may be reversed under the possibility of tax evasion, let us rewrite the change of profits due to a marginal tax rate change in generic form as

$$\frac{d\pi}{dt} = \overset{-}{\frac{\partial \pi^{NE}}{\partial t}} - \frac{m}{2}\frac{\overbrace{\partial\left(pq-d\right)^2}}{\partial t} - (1-m)\overset{-}{\frac{\partial\left(td\right)}{\partial t}} \tag{16}$$

Eq. (16) decomposes the total effect of a change of the tax rate on profit in three parts. The first term represents the standard negative effect on profit which would work in the case of the absence of tax evasion (π^{NE}). The second term represents the negative effect on profit due to the expected penalty (this effect is magnified by an increasing detection probability). The third term is the positive effect on profit due to the increase of tax evasion (this effect is decreasing with the detection probability). Therefore, Eq. (16) shows that if the tax rates are increased, then the gain due to the larger tax evasion may outweigh the sum of the standard negative effect on profit plus the higher penalty, provided that the detection probability is sufficiently low.[12] A remark emerges from the above discussion:

[12] This role played by the detection probability - namely the lower such a probability the more likely profits are increasing with taxation – is clearly illustrated in the next Figures *4.a* and

Remark 2. *Firms would prefer tax policies that, ceteris paribus as regards the level of public revenue, are based on a mix of high tax rates and low monitoring.*

Second, we define tax revenue as

$$T = 2\left[m(tpq + \frac{(pq-d)^2}{2}) + (1-m)td \right] = -\frac{t\left[(1+m^2)tk + 2m\left[a^2(1-t)^2(\gamma-1) + c\gamma a(t-1) + c^2 - tk \right]\right]}{mk} \tag{17}$$

By observing (17), we note that, although it is possible that a Laffer-curve type phenomenon for the tax revenue may occur when tax rates become sufficiently high, tax revenue is increasing with increasing tax rates[13] in most situations in which the unconventional positive tax effect on profits emerges, as shown in Figure 2 by the joint illustration of both the relationship between revenue and profits, on the one hand, and tax rates, on the other hand. Such a figure shows that: 1) profits are increasing with increasing tax rates when detection probability is sufficiently low (see the parametric regions A and B); 2) revenue is increasing with increasing tax rates when tax rates are sufficiently low (see the parametric regions A and D); 3) thus in the region A both profits and revenue are increasing with increasing tax rates.

4.b, in which it is shown how profits behave when tax rates increase for the cases of *m=0.15* and *m=0.30*.

[13] The "humped" shape of the relationship between revenue and tax rate holds provided that marginal costs are positive, irrespective of whether evasion is present or not.

Extensive numerical simulations for different values of *a, c* and γ show that the qualitative pattern of the curves and regions depicted in Fig. 2 is invariant to such values, unless *c*, γ, or both together, are sufficiently small.[14]

More in detail, Figure 3 shows that a revenue-maximising Government, provided that to increase the audit effort beyond the detection probability of about 50% is not possible, should set in a sufficiently competitive market (i.e., $\gamma = 0.9$) the optimal values of the couple of policy instruments "detection probability-tax rate" along the segment *x-y* on the revenue maximising curve, where profits are increasing with the tax rate. The case in which a government could prefer the channel of high tax rates instead of a larger effort for auditing firms to enhance its tax revenue is when the latter is very costly and/or auditors are ineffective or corrupted. Thus, the following Remark holds:

Remark 3. *Provided that auditing activities are adequately low, firms could support tax policies of a tax revenue-interested government based on feasible high tax rates.*

This novel result occurs for a large set of values of the couple of policy parameters showed by the region *A* and, more strictly, along the segment *x-y* in Figure 3. Moreover, for the values of the parameters' couple in the point *y* both tax revenue and profits are maximised.

[Figure 3 about here]

[14] In fact when c and/or γ are small, that is, loosely speaking, competition is low, the conventional wisdom that tax revenue and profits are increasing and decreasing, respectively, when tax rates increase, tends to be restored.

While the above analysis and Figure 3 have shown the situations in which higher taxes may enhance both firms' profits and public revenue, we illustrate the content of Rem. 3 showing that firms may even sustain that the government sets the highest possible tax rates. Figures 4.*a* and 4.*b* show, in a market situation sufficiently competitive (i.e. $\gamma=0.90$), that firms always prefer the maximal tax rates [15] – for which also the tax revenue is always increasing and thus, of course, such rates are preferred by government as well – in the cases with $m=0.15$ and $m=0.30$.

[Figure 4.a about here]

[Figure 4.b about here]

[15] In fact, it is easy to observe that in both figures profits are higher with the maximal tax rate than without taxation. The maximal tax rates are reached when firms choose to hide the entire sales revenue (this formally is only implicitly given by the tax rate for which $d=0$, see Eq. (8)). Of course, for further increases of the tax rate, the positive effect on profits due to the tax evasion ceases to work and then profits are reduced by further tax increases. However, in general (that is, unless the detection probability is very small) this occurs for tax rates values which are higher than those implemented in the real world: for instance, in fig. *3.b* where $m=0.30$, the maximal tax rate is about *48%*. Moreover, note that for higher levels of the detection probability, the maximal tax rate tends to unity, and then intervals including sufficiently high tax rates in which profits are increasing with increasing tax rates exist also for higher values of m.

Finally, let us see whether the tax policy and evasion activities impacts on social welfare. The social welfare function, SW, can be defined as the sum of the firms' (expected) profits, consumer surplus, $CS = \left(\frac{q_1^2 + 2cq_1 q_2 + q_2^2}{2} \right)$, and government (expected) tax revenues:

$$SW = \pi_1 + \pi_2 + CS + T \, .$$

Making use of (9) and (12) and (17), one obtains

$$SW = \frac{-(a(1-t)-c)\{-a(1-t)(3+c-2\gamma^2)+c[c+2t(\gamma^2-\gamma-2)-2g^2+3]\}}{(1+g)k}$$

which is an expression independent of the detection probability of indirect tax evasion activities, and with the standard comparative statics $\frac{\partial CS}{\partial t} < 0$ and $\frac{\partial SW}{\partial t} < 0$. In other words, auditing activities play the role of shifting (expected) resources between government and firms while keeping unaltered consumers' well-being.

From the overall above analysis, the conclusion is that, especially in those countries vitiated by inefficient and expensive audits and corruption of auditors where tax policy is mainly based on tax rates, a Leviathan Government may, pursuing exclusively its interest, also indirectly increase firms' profits, so obtaining the political support of firms' owners.

3. Conclusions

This work contributes to the literature on tax compliance by firms, focusing on a so far unexplored issue, namely the relationship between profits and taxes in a price competition context. In particular, it investigates whether the conventional wisdom that indirect (sales) taxes penalise profits may be challenged in the presence of tax evasion by firms. To analyse this issue, the tax compliance behaviour in a duopoly with differentiated products under price competition is studied. In contrast to the preceding literature, it is shown that indirect taxes may increase firms' profits. This unconventional finding is more likely to appear when there exists a sufficient competitive pressure in the industry and the effort for detecting evasion is not too strong. The policy implication is that firms could support tax policies that not only, ceteris paribus as regards the level of tax revenues, are based on a mix of high taxes and low audit probability, but also - provided that the latter is sufficiently low- on as high as possible tax rates. Therefore, since public revenues always increase with tax rates, government and firms may agree on high tax rates which may enhance both profits and public revenues. This result may be important for those countries with a Leviathan Government which cannot improve its revenues through the detection of evasion because audits are very costly and/or auditors are ineffective or corrupted. For instance, there is some empirical evidence that, even in developed countries with strong institutions, Governments with the political support of big firms and corrupted tax administration coexist. Indeed, evidence of this link between corruption and tax evasion by firms is documented in Alm et al. (2016). Firms pay bribes when dealing with taxes, and for three-quarters of countries in the sample considered in this study, bribes account for 0.5 percent of sales. Higher incidence of bribes is associated with lower proportion of reported sales. The model in this paper does not embed explicitly the corruption mechanism; in fact, the probability of audit is exogenous, and the model disregards the possibility of weak monitoring being a result of corrupted tax officials. An extension worth to be further investigated would be to include explicitly in the model the amount of bribe (for example, as a proportion of sales) that would reduce the probability of audit, such that the model's predictions could be then compared to the empirical findings in Alm et al. (2016).[16]

A few other extensions of the current work can be suggested. Analysing a Cournot duopoly is a natural extension.[17] The analysis of different tax systems such as specific and corporate taxes as well as different industry structures such as vertical relationships is also of interest.

Conflict of Interest: the Authors declare that they have no conflict of interest.

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[16] We thank an anonymous referee for having signaled this empirical contribution.

[17] By passing, we note that Katz and Rosen (1985) argue that, in the case of the conjectural variation parameter about zero corresponding to the Cournot competition, the conventional view that profits are always reduced by taxes holds.

Appendix

Proof of Proposition 1

The denominator of (13) is unambiguously positive. Denoting the numerator of (13) as K, it is easy to see that $\lim_{t\to 0} K = m(c^2 - a^2)(1-\gamma) < 0$ and $\lim_{t\to 1} K = c^2 m(1-\gamma) > 0$,[18] that is the relationship is negative (positive) for very small (large) values of the tax rate. This also suggests the presence of a minimum for the function K in $t \in (0,1)$. Indeed, $\frac{\partial K}{\partial t} \leq 0$ if $t \leq \frac{(1-m)^2 \gamma^2 (\gamma - 3) + 2a^2 m(1-\gamma) + 4(1-m)^2}{(1-m)^2 (1+\gamma)(2-\gamma)^2}$, which implies that, to assume values in the range $t \in (0,1)$, the market size has to be included within the range $a \in \left(\frac{(2-\gamma)(1-m)\sqrt{2m(1+\gamma)(\gamma-1)}}{(1-\gamma)m}, \frac{(2-\gamma)(1-m)\sqrt{m(1+\gamma)(1-\gamma)}}{(1-\gamma)m} \right)$. Analytical inspection reveals that the lower bound of that range takes always imaginary values for $m \in (0,1)$ and $\gamma \in (0,1)$, which means that $t = 0$ is never a minimum for K (and therefore, it is never possible that $\frac{\partial \pi}{\partial t} > 0$ in $t \in (0,1)$), while $t = 1$ is a minimum for K if and only if $a = a^{\max}$

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[18] However, $\lim_{t\to 1} K\big|_{c=0} = 0$. The rationale for this result can be seen as follows. Equations (7) and (9) can be rewritten in the following way:

$$p_i = p_j = p = \frac{a(1-\gamma)}{(2-\gamma)} + \frac{c}{(2-\gamma)(1-t)} \quad (7.\text{bis})$$

$$q_i = q_j = q = \frac{a}{(1+\gamma)(2-\gamma)} - \frac{c}{(1+\gamma)(2-\gamma)(1-t)} \quad (9.\text{bis})$$

from which it is evident that taxation affects equilibrium quantities and prices through the term in which marginal costs appear.

(and therefore, $\frac{\partial \pi}{\partial t} < 0$ in $t \in (0,1)$). Those conditions ensure that the function K has only one change of its sign in the interval of interest $t \in (0,1)$. *Q.E.D.*

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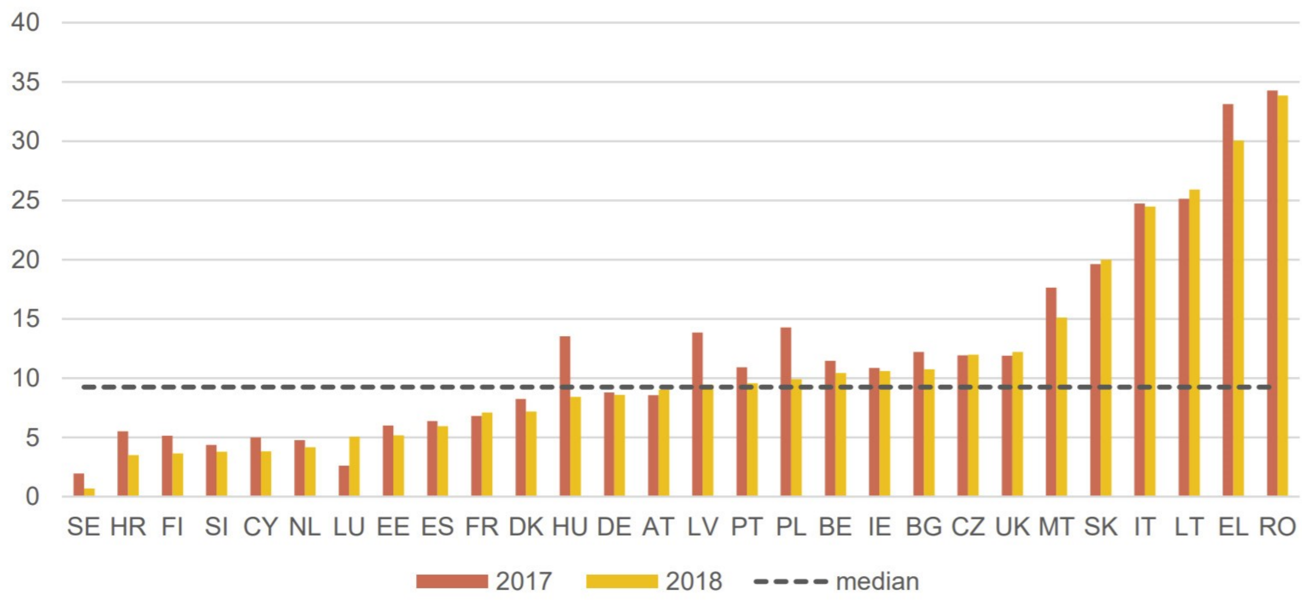
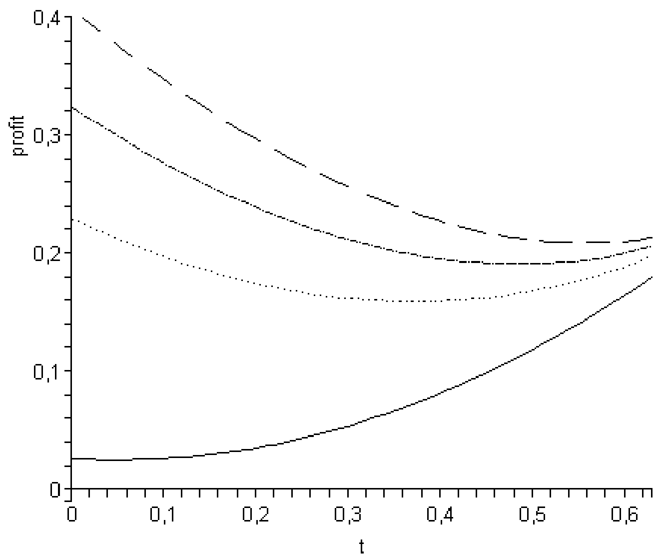
Figure 1. *VAT Gap as a percent of the VAT total tax liability in EU-28 Member States, 2018 and 2017.*



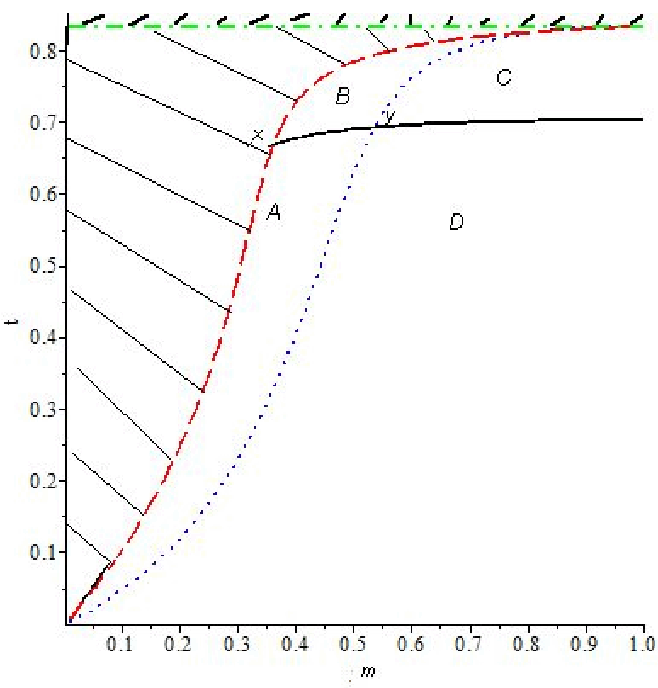
Source: European Commission (2020b).

Figure 2. *Profits as function of the tax rate ($t \in (0, 0.63)$), for different degrees of product substitutability: profits with γ=0.99 (solid line), profits with γ=0.90 (dotted line), profits with γ=0.85 (dashed-dotted line) and profits with γ=0.80 (dashed line).*



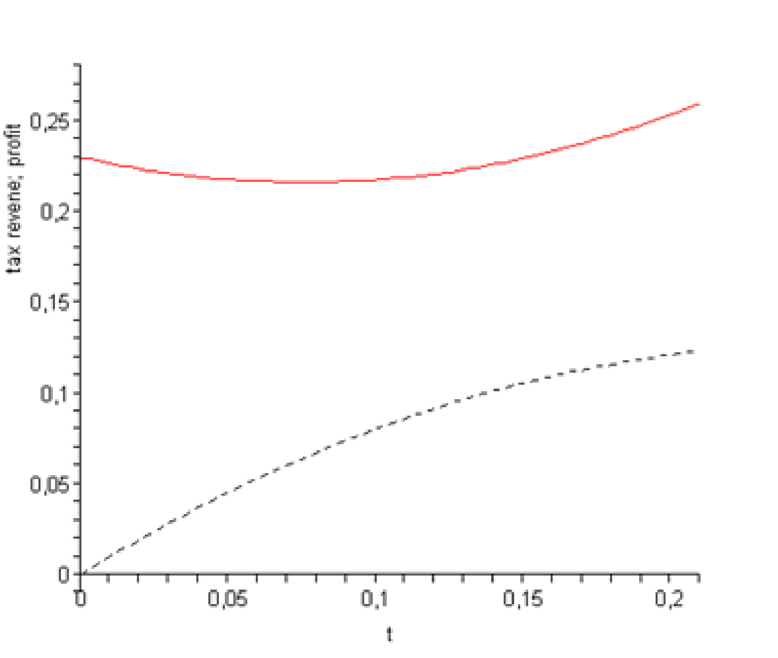
Source: authors' calculations.

Figure 3. *Curves along which 1) it is maximal the value of profits (dotted line) and tax revenue (solid line), 2) the declared tax base is zero (dashed line) and 3) quantities are zero (dashed-dotted line). Legend: i) Parametric set:* $a = 3; c = 0.5; \gamma = 0.9$ *. ii) Region A:* $\frac{\partial \pi}{\partial t} > 0,\ \frac{\partial T}{\partial t} > 0$ *; region B:* $\frac{\partial \pi}{\partial t} > 0,\ \frac{\partial T}{\partial t} < 0$ *; region C:* $\frac{\partial \pi}{\partial t} < 0,\ \frac{\partial T}{\partial t} < 0$ *; region D:* $\frac{\partial \pi}{\partial t} < 0,\ \frac{\partial T}{\partial t} > 0$ *.iii) Shadowed regions are economically unfeasible because on the left of the dashed line, the declared tax base would be negative and above the dashed-dotted line quantities would be negative.*



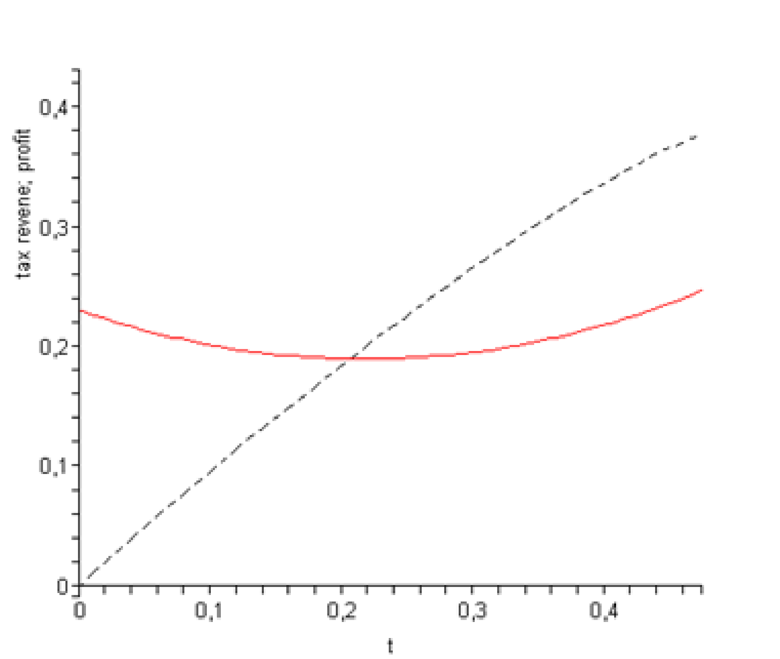
Source: authors' calculations.

Figure 4.a *Tax revenue (dotted line) and profit (solid line) as function of the tax rate ($t \in (0, 0.205)$), with $a=3$, $c=0.7$, $\gamma=0.90$ and $m=0.15$.*



Source: authors' calculations.

Figure 4.b *Tax revenue (dotted line) and profit (solid line) as function of the tax rate ($t \in (0, 0.475)$), with $a=3$, $c=0.7$, $\gamma=0.90$ and $m=0.30$.*



Source: authors' calculations.